Mathematics Learning and Teaching in an Interdisciplinary Framework Simulating Ancient Academy

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Introduction

A scientific revolution arose during the seventeenth centuries, also thanks to the development of new institutions for the acquisition and dissemination of knowledge. The most important of these new institutions were the Academies (Ruscelli et al., 1984). Over 800 Academies flourished in Italy in the period 1525-1700, forming a significant and influential aspect of social and intellectual culture. Interdisciplinary in their interests, bridging literature, arts, medicine, and sciences, the Academies operated outside, but were often interconnected with official institutions like universities, courts, political and religious bodies, and offered a more flexible, apparently free and equal form of association. Members or affiliates could sometimes include socially marginal figures like women and artisans. Academies attracted also foreign intellectuals and their networks extended across Europe.

In the present work, a new learning/teaching approach has been introduced, authors named “Academic Education”, to learn Mathematics and Science in an interdisciplinary framework, as in an ancient Academy, but with innovative and effective instruments. Role playing, and drama mediators (Damiani, 1993) have been preferred, to better live the Academic atmosphere.

An interdisciplinary educational path has been planned, in which “hidden” (i.e. absent on the schoolbooks) synthetic Geometry theorems have been re-discovered by simulating scientific Academies. This unit has been experimented with about ninety students, attending second year of a Secondary School.

Information and Communication Technology (ITC) has been used, in particular social networks and dynamical Geometry software GeoGebra.

Main interest in Geometry is because emerging data about study of this branch in literature. In particular, researchers have long reported the difficulties encountered by students of Geometry, referring to such basic geometric concepts as the angle,

triangle and quadrilateral (Gal, 2010). Nevertheless, Özerem (2012) revealed that 7th grade secondary school students have a number of misconceptions, lack of background knowledge, reasoning and basic operation mistakes at the topics: measures, angles and shapes, transformations and construction and 3-D shapes. Moreover, EIA TIMSS2015 data (Jones et al., 2015), show that Italian 8th grade students don't attribute to Maths study an operational value, they think that Maths is not fundamental for their future and their pleasure in the studying Mathematics is very poor.

Main idea is increasing pleasure and motivation in studying Mathematics and Science, creating a social, free scheme environment, starting from free discussions and investigations, not necessarily connected to textbooks, starting from the particular Italian historical background.

Our research starts from these questions:

May a methodological approach based on a social comparison between students improve their motivation in studying Mathematics?

May a methodological approach based on an historical, students self-managed, research intrigue and arouse interest in Mathematics, and especially in Geometry?

May the use of an informal language encourage the students to discuss about Mathematics?

Italian Academies

The idea of "Academy" goes back to ancient Greece: the first Academy was founded by Plato in 387 B.C. in Athens. Aristotle studied there for twenty years before founding his school, Lyceum. Two women are known to have studied with Plato at the Academy, Axiothea of Phlius and Lasthenia of Mantinea. The subjects of study almost certainly included Mathematics as well as the Philosophical topics, but there is little reliable evidence about it.

Traditionally, an Academy has been defined as being composed of a group of individuals interested in intellectual or cultural matters, which held regular meetings to discuss topics of intellectual, cultural or current interest and to promote lectures, dramatic performances, scientific enquiry and experimentation, and to produce publications arising out of these.

In Italy, in the fifteenth century, this idea was revived and was used to refer to groups of humanists who gathered together informally to discuss matters of literature and philosophy arising out of the continuing revival of classical culture, to which the terms Humanism and Renaissance were later applied.

The birth of the Italian Academies in the full sense, as places to disseminate interdisciplinary learning, can be dated to the formation in 1525 of "Intronati Academy" in Siena. The Italian Academies of the sixteenth and seventeenth centuries, thus formally constituted, were the earliest of such institutions, which were subsequently found throughout Europe, as other countries sought to emulate this model. D'Alembert, in his l'Én-cyclopédie, wrote about the word.

An innovative collaborative project (since 2009 up to now) involving Royal Holloway, British Library, and University of Reading about Italian Academies 1525-1700 has been developed and a database has been realized (Italian Academy Database, IAD), which mapped the Italian world of Academies as a global phenomenon, http://www.bl.uk/catalogues/ItalianAcademies/, connecting Academies, books, and people (587 Academies, 7100 People, 911 Works). This project showed that Italian Academies could be considered as the first intellectual "social networks" of early modern Europe (Testa, 2015).

Their membership included pioneering scientists, literary polemicists, political thinkers, women as well as men, and representatives of all social classes. In addition to their intellectual pursuits, the Academies had a more playful aspect, including the delivery of orations based on paradoxes, the performance of games, or the invention of amusing names for the Academy, its members and its activities, often represented visually in punning illustrations and devices. The individual nicknames taken by members reflected the idea present in the name of the Academy. Members of Academies frequently published, for many different reasons, only under their Academy nicknames – and this fact constitutes one of the principal difficulties in researching Academy publications through standard, author-name catalogues.

Varying in size and importance, Academies were of two major types: some preferred to keep a low profile and gathered in privately maintained spaces, others became symbols of ostentation and often met to publicly celebrate courtly life, patrons, and events of both a secular and religious nature. Nevertheless, some common features may be found in all the Academies: an effective name, a logo, a motto, generally a noble or a celebrity who favoured its growth, interdisciplinary discussions and thoughts freedom. Often the place where Academies carried out their activities were unknown and sometimes, they corresponded to private houses.

Irace and Panzarelli (2011) show that in Naples, there were 161 Academies, but 99 places are only known nowadays (53 in private houses, 33 convents). The interests of Academies ranged very widely across the disciplines, from art and literature to the experimental sciences. A study on Academies in Bologna, Florence and Naples in Irace and Panzarelli (2011) analyses most relevant topics in the Academies: 7,7% was about Science and Maths.

First scientific Academy, “Academia secreta”, was founded in south of Italy (probably in Salerno, near Naples) by Girolamo Ruscelli, although few news about it can be found in literature (Ruscelli et al., 1984) and the place where it was born is still unknown.

In scientific Academies, some “geometric problems” were also discussed, taking into account both calculus and synthetic geometry solutions. Few details may be found about these “discussions”, also due to the general academic rules: the members could only have access to all the research material developed inside the Academy.

Academic Education: A New Learning and Teaching Methodological Approach

Starting from main characteristics of Italian Academies, a new learning and teaching methodological approach has been developed, authors named “Academic Education”.

Main features of this approach can be summarised as follow:

- A “new learning place”, an Academy, have to be created, also by choosing all details to identify it (logo, motto, wearing members, eventually rules). This place may be either virtual or real. Virtual Academies could connect students in all the world. Nevertheless, it should be better to context this place, also taking into account the surrounding territory.
- All the students have to be members of an Academy. More than one Academy may be “founded” in a class (or in an open classes group); in this case, heterogeneous groups have to be created, in agreement with ancient Academies members, just paying attention that females should be equally distributed into all the Academies.
- Social networks have to be used as a vehicle to disseminate learning:

- in a “free” way;

- in an anonymous way, depending on the student’s opinion, by using a pseudonym (nickname);

- with a “limited open access”: only the “members” may have a free access to discussions.

- The educational path should be interdisciplinary, focused around a main theme, but also including topics which could emerge from debate (both live and/or virtual debate), not necessary planned by the teacher.
- Virtual meetings should be organized on social networks, either synchronous (planned as in a forum) or asynchronous (chat).
- Live meetings in the blended Academy have to be organized together with students and teachers of different subjects, to give main effort to the interdisciplinary path.

In order to apply this new methodological approach, a scheme “step by step” is reported in Table 1

Table 1. Centre the Caption Above the Table

Action	Time	Teacher Role	Students Role	Observations
Choice of **“Academy typology”: blended or virtual**	1 hour	Teacher takes a decision, depending on: time, topic to be experimented, number of students		- Blended is preferred if topic is hard, time is lower 10 hours, experimentation is limited to one classroom - Virtual Academy is preferred if topic is standard, time is over than 10 hours, experimentation is in open classes
Choice of **the social network**	1 hour	Teacher shows some “didactical” social networks, as Padlet, Edmodo...	Students suggest the most popular social networks they use (Facebook, WhatsApp)	In order to choose the “right” social network, it is important to take into account all the technical characteristics, e.g. possibility to share files, links, maximum dimension of the shared files, the opportunity to create closed groups.
Creation of the “Academy”	1 hour	Teacher and students choose, together, main characteristics of new Academy: name, logo, motto, eventually some rules.		
Choice of **main topic**	1 hour	Teacher chooses main topic and introduces it on the social network, just posting an image or a sentence, with no further information..	Students search on the web and/or on books/journals some information about main topic	The teacher introduction have to be effective, also to engage all the students and to arouse curiosity.
Virtual Debate	3 hours (this time is indicative, it depend on the topic)	Teacher takes part to the discussion only if the students ask for him.	Students share information between them on the social network.	Shared information could be partially mistaken or in contrast between them. Teacher does not “reveals”, at this step, the mistakes
Planning/ **realizing learning activities**	3 hours	Teacher critically analyses student’s discussion and suggests some learning activities to “confute” or “prove” student’s idea	Some students carry out the activities suggested by the teacher and share the results on the social network	The criticality of this step is that not necessarily all the students actively carry out the “assigned works”. Teacher has an immediate trace about it, but he hasn’t to force students.
Extend topics in an Interdisciplinary Framework	3 hours	Teacher suggests some topics, connected to the main theme, but in different contexts (history, art, science, music and so on)	Serendipity gives the opportunity to extend main topic to new and interesting topics, in an interdisciplinary framework	Two experimentations of the same path could be very different between them, depending on this step. At this step, further teachers of different subjects could be added to the “Virtual Academy”
Live Meeting **(only in case of blended Academy)**	5 hours	Teachers of different subjects actively attend to the discussion	Students show their research and activities results; organize their own learning unit.	ICT are fundamental to better organize learning unit, nevertheless also traditional instrument can be used
Final product	2 hours	Teacher suggests realizing a final product, s	Students decide modality to realize final product	Final product should be: Multimedia, Easy to share, Effective, Interdisciplinary

A case study: The Baffling Academy

Taking into account the "Academic Education", an interdisciplinary educational path has been planned. Learning unit has been experimented with about ninety students, attending the second year at High School; times is in agreement with Table 1 (about 20 hours). The final live meeting (5 hours) has been realized together with History and Literature, Latin, Science, English and Maths teachers.

Students used their smart-phones and/or tablets, as in Bring Your Own Device (BYOD) practises, whereas a LIM and a PC have been used in the classroom, too. In order to verify Geometry theorems, interactive and dynamical geometry software GeoGebra, has been used. Historical texts have been also consulted, mainly on the web. Let's summarize experimental phases, following steps in Tab.1. Here we show results about a second class of a Scientific High School, composed by 27 students.

Choice of Academy typology

We created both types of Academies:

a) three virtual Academies, in which members communicate between them only by using social networks;

b) a blended Academy, with both virtual and live meetings. The motivation of this choice will be clarified later in the experimental details.

Choice of Social Network

After a debate between students and teacher, *WhatsApp* has been selected as social network, to create "closed virtual places", where they met and had their learning activities.

Choice of Main Topics

Maths teacher suggested, as main topic, "synthetic Geometry theorems and problems on triangles, which are missing in Maths schoolbooks, but largely diffused on the web (blogs, Maths Forums, Wiki, Youtube)".

Creating 2.0 Virtual Academies

At the beginning, three virtual Academies have been created (in this specific case, 9 students in each group), named as their specific research theme:

- Cevian Triangles
- Pedal Triangles
- Orthic Triangles

For each Academy, students chose: a logo, a motto, some communication and sharing rules. The "WhatsApp ImageGroup" was the Logo, chosen in agreement with all the members (which were all administrators, so nobody was the leader). Teacher was also added to each Academy group. Students used their nicknames to login on the social network.

An introductive message was posted in each Academy/group by the teacher, suggesting main topic, as an example:

"27/10/17, 18:14 Hi guys, now you are Pedal Academy members. Search on the web and post here all you can find about pedal triangles, both historical and geometric aspects".

Informal language has been used by the teacher, too.

Virtual Debate - Planning and Realizing Learning Activities

Virtual members posted on the groups their web researches: video, images, idea, suggestions and comments. Teacher suggested some learning activities.

Communication was asynchronous, students felt free to have their research in each place and time, so this new peer to peer - cooperative e-learning was effective and students feel enthusiast.

Just as an example, let's analyse a brief virtual dialogue, between students and teacher:

Pedal Academy

27/10/17, 18:28 - Mirko: I found and interesting link http://web.mclink.it/MC2113/geometria/java/Tpedali.html
28/10/17, 19:02 Paolo "I found definition of pedal triangle: in Geometry a pedal triangle of a point with respect a triangle is identified by the point projections on the triangle sides."
[...]
05/11/17, 19:29 - Teacher: "Question: May I choose all the points I want?"
05/11/17, 19:42 Andrea: "Yes you can, orthocentre, incentre and circumcentre, too"
05/11/17, 19:43 Gabriele: "No, you can't choose all the points, they have to be inside the triangle"
05/11/17, 19:47 Mirko: "I agree with Gabriele, just for an example, I think that the circumcentre of an obtuse triangle is out of the triangle, so I think it isn't OK."
Andrea: used an emoticon image to say "I LIKE"
05/11/17, 20:05 Teacher: "Have you tried to represent it with some special points?"
Andrea posted 3 Geogebra files (as you can see in the Topics section)
05/11/17, 20:12 Teacher: "You have just studied circumference, look if there is a

connection between pedal triangle and circumference."
05/11/17, 20:14 Gabriele "Prof, I'm trying... I draw the pedal triangle of the orthocentre (D), by Geogebra, I draw a circumference passing by D and a vertex C, CD is a diameter...
05/11/17, 20:20 Andrea "Gabriele, I put the Pedal of the Incentre, CD is always the diameter..."
05/11/17, 21:05 Giampietro "Diameter is perpendicular to the cord, it is obvious"
Teacher "Wonderful"
05/11/17, 21:50 Paolo: "I'm searching for historical origin of pedal word, but I didn't find anything...who invented this word?"

By analysing the web debates, we may answer to some research questions:

Students discuss about Mathematics between them, without any fear to make a mistake. Students use an informal language, also including "emoticon" to approve or disapprove, as in a friendly chat.

In this free scheme framework, students have the opportunity to do Maths research and rediscover some results, which are not present in their schoolbooks, but are strictly connected to standard geometric topics.

Students are very interested in the topics, so they continue to study Maths also in the evening. In the research phase, GeoGebra has a relevant role, as a flexible instrument to immediately verify some student's hypothesis and some theorems they found on the web.

Extended Topics

In order to better comprehend the context where this new methodology has been experimented, teacher suggested also, in each virtual Academy, to get information about some specific ancient Academies founded in Campania, which are connected between them, because founder of one Academy was also a member of another one, and so on. It is just a way to "follow a branch" of the complex ancient Academies network.

Some Criticisms in Virtual Academies: towards Blended Academies - Live Meetings

Students had some problems to virtually organize their web researches in a learning unit: a virtual discussion to individuate main elements and synthesize them in a learning unit was not trivial.

For this reason, all the students agreed to change "virtual Academies" in "blended" ones: members met at school, discussed about their topics and each Academy produced a poster, to be shown to the members of other Academies (e.g. see Figure 1 for the Cevian Poster).

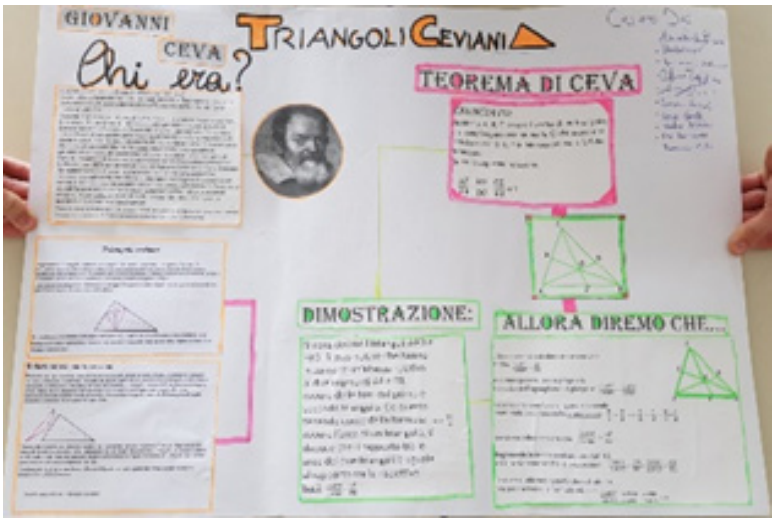

Figure 1. Poster realized by Cevian Triangles Academy. Similar posters have been realized by Pedal and Orthic ones.

A debate (Leron & Hazzan, 2006) between all the students belonging to the three Academies showed that there were some common features in the analysed topics.

Because of these similarities, and in order to realize a more complete learning unit, the three Academies fused together to create a new one, named *Baffling Triangles Academy*. Re-following the first steps, all details about this new Academy were established: the motto was "T*riangles: a universe letter"*, inspired to the Galileo's words. A uniform was suggested in a black sweatshirt, possibly with a hood.

A day trip to Langobard old town Salerno (a few chilometers far from the school) was also organized to better context the research activities, in particular to rediscover the place where the first Scientific Academy, Academia Secreta, (Ruscelli, 1984) was probably founded. Students were mainly fascinated by a specific geometry, releaved on the floor in San Pietro a Corte church (see Figure 2) and they chose it as Academy logo.

Figure 2. A mysterious Geometry on the floor in San Pietro a Corte (Salerno, South of Italy).

Final Live meeting

Final live meeting of Baffling Academy was at the presence of all the students and teachers of different subjects (Maths, Latin, English, History and Literature, Science). Students showed main results about all the topics, also detailing the used historical sources and the ICT instruments they preferred.

Students used three different languages: Latin (for some historical fonts, i.e. Ceva), Italian (but often archaic italian) and English (e.g. Coxeter and Greitzer, 1967). They showed their results, both by using multimedia and posters. A debate between teachers and students was effective in order to give effort to the interdisciplinarity of the learning unit.

Final Product

Finally, students realized a multimedia product (video), summarizing their own point of view about this educational path. They also added some comments, which were similar to slogans, i.e.:

"From the past to the future: we worked as Academy members"
"Our Academies 2.0: comparison, dialogues and research"
"Learning all together is special!"

By analysing students' slogans, we deduce that a strength of this methodological approach is the free cooperative comparison and research spirit: students feel part of an organization where they may discuss, without no fear to have a mistake.

Topics

Analysed topics are Geometry theorems about triangles, discovered after 1500, which are not classified, not well dated, not always organized, not present in Maths school books, but they are very useful, interesting and easy to be studied at High School. Here we show just some students researches from the three virtual Academies:

Cevian Triangles

A Cevian is any line segment in a triangle with one endpoint on a vertex of the triangle and the other endpoint on the opposite side.

Given a point S and a triangle ABC, the Cevian triangle PQR is defined as the triangle composed of the endpoints of the cevians though the Cevian Point S (see Figure 3).

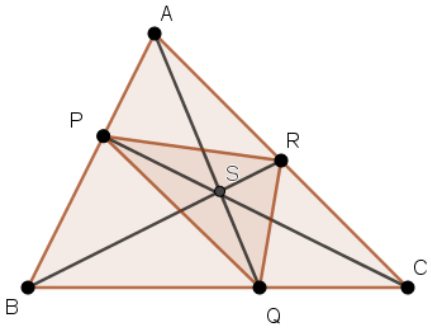

Figure 3. PQR: Cevian Triangle of the Main ABC Triangle, S = Cevian Point.

The condition for three general Cevians from the three vertices of a triangle to concur is known as Ceva's theorem.

Giovanni Ceva studied Geometry for most of his long life. In 1678, he published "De lineis rectis se invicem secantibus statica constructio", about applications of mechanics and statics to geometric systems, including the new theorem on synthetic Geometry in a triangle, known with his name.

Theorem 1 (Ceva's theorem)

In a triangle ABC, three lines AQ, BR and CP intersect at a single point S (i.e. they are concurrent) if and only if:

$$\frac{AP}{PB}\cdot\frac{BQ}{QC}\cdot\frac{CR}{RA}=1$$

Students verified it by using GeoGebra (See Figure 4)

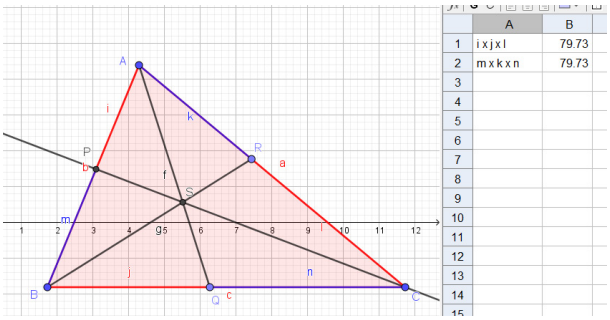

Figure 4. Test of Ceva's Theorem by Using GeoGebra.

Ceva's theorem is very important in Geometry, because it is simple to prove that medians, angle-bisectors, and heights are Cevians, i.e. they all concur in a Cevian point (barycentre, incentre and orthocentre points, respectively).

Students verified these properties by using GeoGebra and they found on the web some different proofs about it, also by looking for some Youtube videos, which they believed

were effective and very simple.

Maths directly studied on Youtube channels is a further response to our question research: it can be considered as an instrument which is free, closer to their attitudes and it brilliantly substitutes a long, boring chapter about “Remarkable Points in a Triangle”.

This theorem is also important in Physics, in particular in the Mechanics and a debate about its physical aspects has been also done.

Pedal Triangles

A pedal triangle is obtained by projecting a point onto the sides of a triangle.

More specifically, let’s consider a triangle ABC, and a point P which is not one of the vertices A, B, C. Let’s drop perpendiculars from P to the three sides of the triangle (these may need to be produced, i.e., extended) and label L, M, N the intersections of the lines from P with the sides BC, AC, AB. The pedal triangle is the LMN one (see Figure 5)

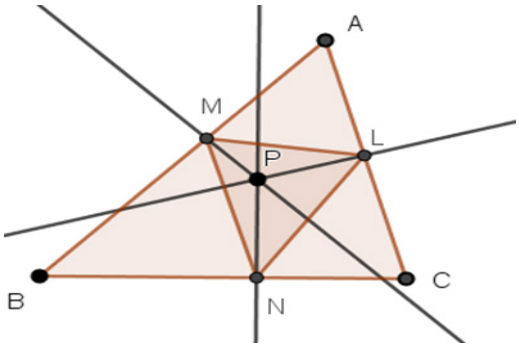

Figure 5. LMN: Pedal Triangle of Main Triangle ABC.

Several properties can be proved about pedal triangles, all starting from a “main property”:

Theorem 2 (Main Pedal Property) Given an ABC triangle and an $A_1B_1C_1$ pedal triangle respect to P point, then A, C_1, P and B_1 belong to a same circumference, with AP as a diameter” (Figure 6).

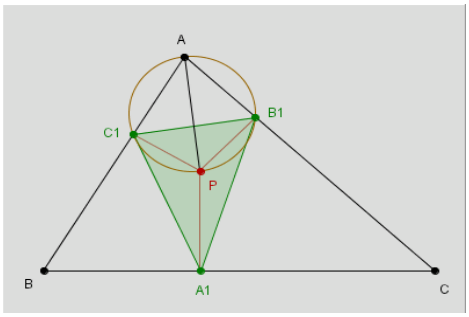

Figure 6. Pedal Triangles Main Property: A, C_1, P and B_1 belong to a same Circumference, Being AP the Diameter

Proof

Trivially, from the Pedal definition, angles $AB_1P = AC_1P = 90°$ so both the triangles AB_1P and AC_1P are in a semi-circumference and the quadrilateral AB_1PC_1 is inscribed in a circumference, with diameter AP. Let's observe that this theorem corresponds to the "students' discovery" in the dialogue above.

Orthic Triangles

Given a triangle ABC, the triangle $H_AH_BH_C$, whose vertices are endpoints of the altitudes from each of the vertices of ABC is the orthic triangle. There are three cases:

if ABC is an acute-angled triangle, then the orthic triangle is inside ABC;
if ABC is a right-angled triangle, the orthic triangle degenerates in the hypotenuse height;
if ABC is an obtuse-angles triangle, then the orthic one is outside ABC.

In Fig.7 these three cases have been shown, as found on the web.

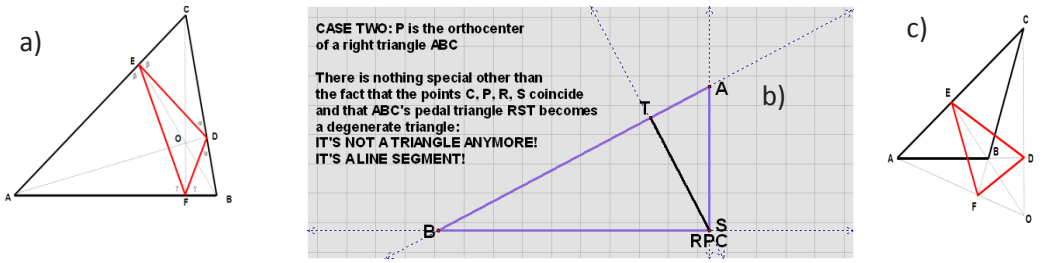

Figure 7. Orthic Triangle in case of a) ABC is an Acute-Angled Triangle, b) ABC is a Right-Angled Triangle, c) ABC is an Octuse-Angled Triangle.

It is important to observe that a bit of confusion can be found on the web, mainly depending on a no systematic arrangement about these topics; for example, in Fig.8.b it is written "Pedal" instead of Orthic triangle" Nevertheless, the Orthic triangle is both the Pedal and Cevian Triangle of a specific point, the Orthocentre.

An important property of Orthic triangle is the following one:

Theorem 3:

If is the orthic triangle of , then ABC heights are DEF angle bisectors, i.e. ABC ORTHOCENTRE is the Orthic triangle (DEF) INCENTRE.

Students verified this theorem in the ITC laboratory, by using GeoGebra, as shown in Figure 8.

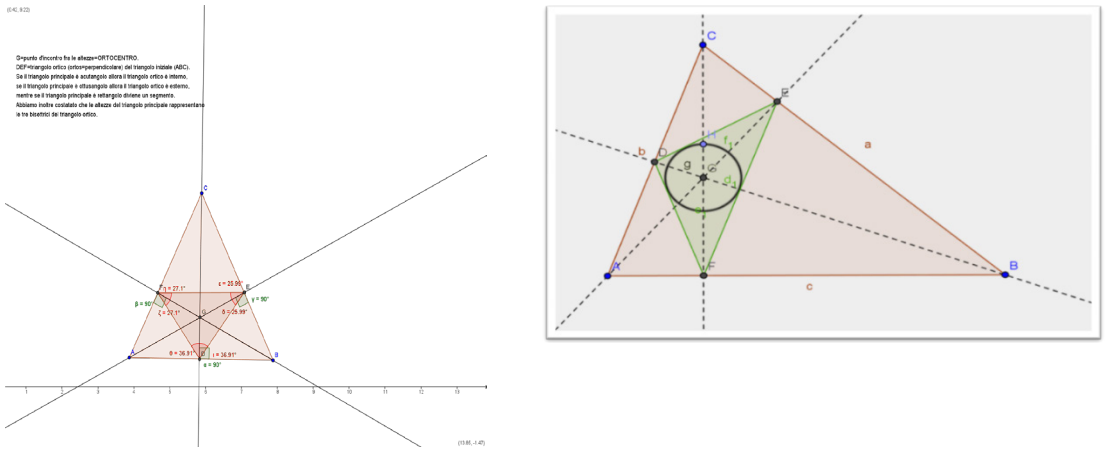

Figure 8. Test of the Property "ABC Orthocentre is the Orthic Triangle Incentre"

During 1700s, because of calculus inception, main discussions were about minimum and maximum problems.

A known minimum problem about these topics was introduced by Giovanni Fagnano in 1775 (Coxeter and Greitzer, 1967):

Theorem 4 (Fagnano's problem)

For a given acute triangle determine the inscribed triangle of minimal perimeter. The solution is "the Orthic triangle". Fagnano' solution used the "calculus", whereas L. Fej´er and While H. A. Schwarz gave a proof by using synthetic geometry (axial symmetries), in an independently way. Students verified Fagnano's problem in the ITC lab, by using GeoGebra, nevertheless, they also analysed the L. Fej´er proof (Coxeter and Greitzer, 1967).

A lot of confusion is still on the web, and orthic and pedal triangles are still mixed up, for the Fagnano's problem, too, as you can see in Gutkin (1997) where it is written "Among all inscribed triangles the pedal triangle has the least perimeter", instead of the orthic triangle.

Conclusion

About 800 Academies were founded in Italy in 1525-1700 years, being a worldwide, very important phenomenon, that introduced a new way to acquire and disseminate knowledge. A new methodological approach, named "Academic Education", has been introduced, starting from the simulation of sixteenth-seventeenth century Italian Academies, all details being specified to apply this approach step by step. Moreover, a case of study has been analysed, by using this methodological approach: an interdisciplinary learning unit has been planned and experimented with about ninety students attending the second year of a secondary school, focused on "synthetic geometry theorems, absent in the Italian maths schoolbooks". Extended topics were

about some ancient Academies in Campania, which were connected between them as in a social network.

By analysing students protocols, we deduced that Academy simulation gave a considerable boost to the Maths social use, which is very important to increase motivation to study scientific matters. They used and informal language on the social networks, and we observed that it encourage them to discuss about Mathematics out of the school context. Students feel enthusiastic about this new learning of way and their feeling were also evidenced in a final video they realized to summarize all the activities about this learning unit.

Moreover, historical aspects both about historical fonts of some theorems and researches about ancient Academies increased interest and curiosity in studying Geometry.

The use of ICT has been fundamental to immediately have a verification of all the Geometry properties.

Nevertheless, the use of web instead of schoolbooks could be a bit misleading, because of some conflicting information have been found on the net.

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