A REVIEW OF RESEARCH ON THE MISCONCEPTIONS IN MATHEMATICS EDUCATION

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ABSTRACT: Misconceptions have been determined as one of the most important bariers on learning mathematics. In this study, it is aimed to investigate and review the articles about misconceptions in mathematics and for this reason conceptual review method was conducted. Within the scope of the study, 21 articles published between 2004 and 2015 were selected through pre-determined criteria. Findings of the review revealed that the number of studies on mathematical misconceptions has increased in the last 5 years. In addition, most of the studies were conducted with primary, elementary and high school students. In these studies researchers generally used multiple choice or open-ended achievement tests. Moreover, most of the studies were conducted for the purpose of determining misconceptions, not eliminating misconceptions. Lastly, some recommendations are provided related to findings of the study.

Keywords: misconceptions, mathematics, concept learning

INTRODUCTION

Progression in educational settings has recently led us to give importance to learning concepts instead of teaching subjects (Posner, Strike, Hewson & Gertzog, 1982). Concept learning begins when the individual comes to the world and continues till the end of life (Ülgen, 2001). However, concept learning is given importance in school programs at every grade level, because learning concepts underlies the learning and cognitive development (Senemoğlu, 2013).

Concepts are perceived as building blocks of knowledge (Altun, 2004), so they reflect the attributes of different objects or facts (Eggen & Kauchak, 1997). We use concepts in all parts of our life when shaping our beliefs, knowledge and actions (Elliott, Kratochwill, Littlefield Cook & Travers, 2000) since they can represent the names of events, thoughts or humans that we encounter in our environment (Kaptan, 1999). According to Demircioğlu (2003), concepts grasp the complexity of knowledge and help us to make sense of both physical and social world.

In order to better provide learning concepts, it is important to know students' prior knowledge and develop new strategies that are appropriate with this knowledge (Akdeniz, Bektaş & Yiğit, 2000). Similarly, Ausubel (1968) advocates that existing conceptual knowledge of students on a specific field can make the most significant impact on the conceptual learning of students. Since learning depends on the cognitive level of learners and complexity of concepts that are to be learned, it is not possible to expect that every individual will learn concepts in the same pace or in a correct way (Fidan, 1999). Thus, in this concept learning process, some students may go off the road and procude misunderstandings which are called as misconceptions (Meşeci, Tekin & Karamustafaoğlu, 2013). There are many definitions of misconception in literature. For example, misconceptions are defined as the perceptions that have very different or wrong meaning from experts' opininon on a certain topic or field (Hammer, 1996). According to Ojose (2015) misconception is a kind of misunderstanding and misinterpretation which is derived from inaccurate meanings.

Misconceptions emerge as a result of experiences and wrong beliefs of individuals (Baki, 1999). Every individual has a unique thinking system which is used in sense-making and expressing the world. If these thinking systems are faulty or deficient, they constitute the bases for misconceptions (Mestre, 1987). As Yağbasan and Gülçiçek (2003) state that if the learner understands a concept as fundamentally different from its scientific meaning, then he/she most probably will construct misconception.

In literature, some studies demonstrated that people cannot distinguish the terms misconception and error (Luneta and Makonye, 2010). According to Eryılmaz and Sürmeli (2002) misconceptions are the subset of errors, which means one can define all misconceptions as errors but all errors may not be misconception. In other words, error is the result of misconception, or misconception is a type of perception which systematically produces error (Smith, diSessa & Roschelle, 1993). Therefore, it is more important that teachers should focus on misconceptions which are the source of errors (Eryılmaz, 2002).

Students bring their perceptions, beliefs and thoughts about the world into classroom (Murphy & Alexander, 2004). When students learn new concepts, they build new knowledge on their previous perceptions, beliefs and thoughts.

That is, if any misconception exists in previous concepts, it is highly that new concepts will include misconceptions as well (Baki, 1999; Driver & Easley, 1978). Thus, as educators we need to know the possible reasons that lie behind these misconceptions and take precautions to provide more efficient learning environments (Ojose, 2015). In other words, determining and eliminating students' misconceptions help teachers understand students' background and perceptions of an academic subject and shape their instructional methods (Murphy & Alexander, 2004).

Misconceptions are very important in learning because students learn new things by relating them with their prior knowledge (Driver & Easley, 1978). Furthermore, most of the misconceptions are deep-seated, widespread and cause permanent obstacles for conceptual understanding (Minstrell, 1982). Since misconceptions are consistent and supported by the individuals' experiences, they steadily resist changing (Cox & Mouw, 1992; Karataş, Köse & Coştu, 2003).

Mathematics is one of the disciplines in which concept learning has an important place, thus many misconceptions might be observed in mathematics. In fact, it is almost impossible in mathematics to define any concept without using many other concepts, since mathematics curricula have a spiral attribute (Ersoy, 2006). Hence, students who have misconceptions in previous topics of mathematics can attach new misconceptions to the previous ones (Şandır, Ubuz & Argün, 2007). For this reason, researchers have been conducting studies on determining and finding ways to eliminate the misconceptions in mathematics for many years (Türkdoğan, Güler, Bülbül & Danişman, 2015). When we search the literature, we realized that there was a lack of review studies which investigate the articles on misconceptions in mathematics. Therefore, this study is thought to fill such a gap and be beneficial for teachers at all grade levels.

Purpose and Research Questions

Learning mathematical concepts highly depends on learning previous concepts so it is important to determine and eliminate misconceptions in mathematics. Therefore, the aim of this review is to investigate and synthesize the literature related to misconceptions in mathematics. Although there are many researches on misconceptions in terms of mathematical concepts, the number of studies that synthesize the studies on misconceptions in mathematics is very limited. Therefore, we tried to find an answer to the question "what are the general characteristics of studies related to misconception in mathematics in national and international field?" On the basis of this research question, answers were searched for the following questions:

1- What kind of misconception studies are conducted in mathematics?
2- Which methods/ways are used in order to determine misconceptions in mathematics?
3- Which techniques are used in order to eliminate the mathematical misconceptions?
4- Which learning areas of mathematics do the studies focus on?

METHOD

In this study, conceptual review/synthesis was conducted in order to investigate the studies related to misconceptions in mathematics. Conceptual reviews are the type of reviews in which the researcher aims to provide an overview of the literature in a given field, including main ideas, models and debates (Petticrew & Roberts, 2006). The synthesis consists of three phases throughout the process: i) Determining the inclusion criteria and review procedure, ii) Reviewing the literature iii) Analysis procedure. These phases will be clarified in detail in the next section.

Determining the inclusion criteria and review procedure

In this phase, we decided on the inclusion criteria, keywords and how we will conduct the review of literature. The inclusion criteria consisted of articles that are i) written in qualitative, quantitative or mixed research methods, ii) published in peer reviewed academic journals rather than a technical report, project anecdote or proceedings, iii) written in English or Turkish language, iv) specifically focused on misconceptions in mathematics, and v) being an empirical research. During the search of literature, we used following keywords in different combinations:
Misconceptions, Concept Learning, Mathematics, Misconceptions in mathematics and Turkish meanings of them (Kavram yanılgıları, kavram öğrenme, Matematik, Matematikte Kavram yanılgıları).

Reviewing the Literature

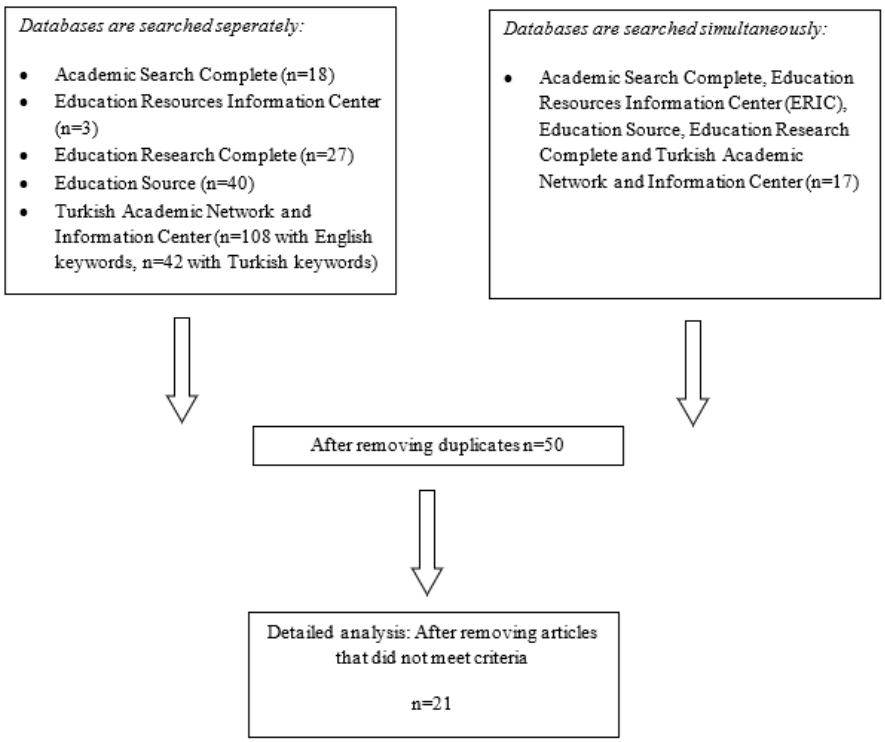
The literature was identified in November and December 2015 by first exploring databases which are more related to educational studies, namely Academic Search Complete, Education Resources Information Center (ERIC), Education Source and Education Research Complete. Then we decided to add database of Turkish Academic Network and Information Center (ULAKBIM) in order to explore literature in the Turkish context.

First, we conducted an advanced search on Academic Search Complete database with keywords "Misconceptions" (in the abstract or Author-Supplied Abstract) and "Concept Learning" (in all text) and "Mathematics" (in the abstract or Author-Supplied Abstract). We filtered the search selecting "academic journals" as source type and "scholarly (peer reviewed) journals". Although there is no limitation according to publication date, the results demonstrated that studies varied between 1997 and 2015. At the end, we reached 18 studies written in English from this search. Second, we conducted another advanced search on Education Resources Information Center (ERIC) database using similar procedures with the previous one. However, this search brought 3 results which were written in English and published between 2003 and 2012.

Next, we continued with another database namely Education Research Complete. Again, we conducted search with the same keywords and limitations. The results occurred 27 articles written in English and published between 1997 and 2015. Then we conducted search on Education Source database using the same keywords and limitations. The results showed that there were 40 studies published between 1990 and 2015. Two of the studies were written in Turkish and remains were in English. Lastly, we conducted an advanced search on Turkish Academic Network and Information Center (ULAKBIM) with the same procedures. In this search we repeated the procedure with the Turkish meanings of keywords. However, the searching engine of this database did not allow limiting the source type. As a result, we reached 108 results when we searched with the English keywords, and we reached 42 results when we searched with the Turkish meanings of the keywords.

After this procedure, we decided to narrow the search down in order to reach the articles that can be more compatible with the inclusion criteria. Therefore, we selected five databases (Academic Search Complete, Education Resources Information Center (ERIC), Education Source, Education Research Complete and Turkish Academic Network and Information Center) simultaneously. Then we searched the literature with the keywords misconceptions (in the title) and mathematics (in the title), selecting peer reviewed and academic journals. Finally, we reached 17 results published between 1999 and 2015.

We investigated all articles obtained from different databases and keywords. Thus, removing the duplicates from the results, 50 studies published between 1990 and 2015 remained. Those 50 studies were subjected to detailed-analysis and we realized that only 21 of them ensured the inclusion criteria determined before. Remains specifically focused on misconceptions in other disciplines (such as science) rather than mathematics, or they were review studies instead of empirical ones. Therefore, we removed these studies from the analyses and focused on those 21 articles.


Databases are searched seperately:
Academic Search Complete (n=18)
Education Resources Information Center (n=3)
Education Research Complete (n=27)
Education Source (n=40)
Turkish Academic Network and Information Center (n=108 with English keywords, n=42 with Turkish keywords)
Databases are searched simultaneously:
Academic Search Complete, Education Resources Information Center (ERIC), Education Source, Education Research Complete and Turkish Academic Network and Information Center (n=17)
After removing duplicates n=50
Detailed analysis: After removing articles that did not meet criteria
n=21

Analysis Procedure

The remaining articles that are selected in terms of inclusion criteria were organized and coded into Table 1 below, according to their subject domain, context, method, data sources, data analysis, reliability, validity/ trustworthiness, etc.

Table 1. An analysis of studies on misconceptions in mathematics (n=21)

Study	Subject Domain	Subjects	Type	Method	Data Sources	Reliability/Validity, Trustworthiness report	Misconception
Green, Piel, & Flowers (2008)	Arithmetic	Preservice elementary teachers	Implemetation (Pre-test post-test Experimental)	Quantitative	20-Item Mathematics survey, pretest-posttest	Instruments' validities were determined by using expert judgements.	Eliminating Misconception
Almog & Ilany (2012)	İnequalitites	High school students	Survey	Mixed method	Questionnaire, Interviews		Determining misconception
Gningue, Menil, & Fuchs (2014)	Algebra	College remedial students	Implemetation (Pre-test post-test Experimental)	Mixed method	pre-algebra test, four achievement tests, final examination, Informal interviews with students, 24-question likert scale survey	Reliability and validity measures were available for neither the pre-algebra nor the algebra posttests.	Eliminating Misconception
Gür & Barak (2007)	Derivative	High school students	Survey	Quantitative	7-Item open ended test		Determining misconception
Gür (2009)	Trigonometry	High school students	Survey	Mixed method	A diagnostic test that consists of seven trigonometric questions was prepared and carried out. Observations were done.		Determining misconception
Akbayır (2004)	Series (Calculus)	Undergraduate students	Survey	Quantitative	10-item achievement test		Determining misconception
Akkaya & Durmuş (2006)	Algebra	Primary students (6-8 graders)	Survey	Quantitative	30-Item multiple choice test called "Algebra Test" is prepared	Alpha reliability of the test is calculated as 0.74	Determining misconception
Baştürk & Dönmez (2011)	Limit And Continuity	Preservice teachers	Survey	Qualitative and quantitative	A questionnaire which composed of open and closed-ended questions, interviews, observations		Determining misconception

Durkin & Rittle-Johnson (2015)	Numbers	Primary students	Survey	Quantitative	Three measures of misconceptions to assess students' knowledge early in instruction on decimals that measured the: 1) prevalence of misconception errors based on response patterns, 2) existence of misconceptions in a more abstract context, and 3) strength of misconceptions using confidence ratings.		Determining misconception
Kaplan, İşleyen, & Öztürk (2011)	Ratio-Proportion	Elementary students	Survey	Qualitative (Case study)	10- item diagnostic test, interviews	Cronbach alpha reliability of the test was found .61	Determining misconception
Keçeli & Turanlı (2013)	Complex Numbers	Undergraduate students	Survey	Quantitative	17-item complex numbers diagnostic test.	For validity: expert opinions, for reliability: cronbach alpha .84.	Determining misconception
Köklü & Topçu (2012)	Quadratic Functions	High school students	Implemetation (Quasi-Experimental)	Quantitative (quasi experiemtal)	Two-tier diagnostic instrument for parabolas, Cabri.	Cronbach alpha reliability of the test was found .63	Eliminating Misconception
Kula & Bukova Güzel (2014)	Limit	Preservice teachers	Survey	Qualitative (Case study)			Determining misconception
Lin, Ko, & Kuo (2014)	Algebra	Preservice teachers	Implementation (Pre-test post-test control group d. Experimental)	Quantitative (pre-test posttest control group design)	The Algebraic Misconceptions Test (AMT) and The Attitudes Toward Computers and Mathematics Teaching Questionnaire (ATCMT)	Feedback received from experts, cronbach alpha is found .86 (for test) and .90 (for questionnaire)	Determining misconception
Lin, Yang, & Li (2015)	Numbers	Elementary students	Survey	Quantitative	50-item web-based two-tier test	Two experts in the field of mathematics education reviewed the WTTT-NS items for face and content	Determining misconception

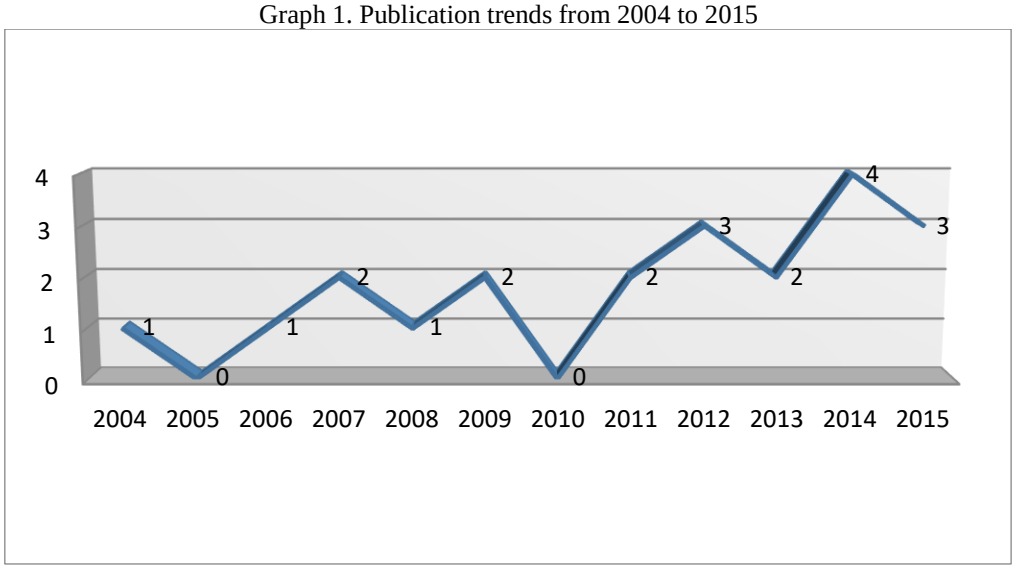
						validity; Cronbach α was found .877	
Muzangwa & Chifamba (2012)	Calculus	Undergraduate students	Survey	Quantitative	10-item test	KR-20 reliability is .31	Determining misconception
Pesen (2007)	Fractions	Primary students (3rd graders)	Survey	Field research and diagnosis method was used	24 item Diagnostic test was used	Class teachers and experts' view to provide validity and reliability of the diagnostic test. Alpha is .90	Determining misconception
Tuna (2013)	Trigonometry	High school (10th grade)	Implementation (Quasi-Experimental with control group)	Quantitative (a quasi-experimental design with control group was used)	14-item test	.	Eliminating Misconception
Türnüklü (2014)	Quadrilaterals	Preservice teachers	Survey	Qualitative (Document analysis)	The data was obtained by the lesson plans which were prepared by the participants	.	Determining misconception
Yang & Lin (2015)	Numbers	Elementary students (5th graders)	Survey	Quantitative	40-item number sense four-tier test	In order to ensure the validity of the instrument, the researchers explained to students how to answer items in the online test system before they started to answer the items.	Determining misconception
Erbaş, Çetinkaya, & Ersoy (2009)	Linear Algebraic Equations	High school students	Survey	Quantitative	56-item Linear equation test	Guttman split half: .95; cronbach alpha: .96	Determining misconception

FINDINGS

Trends in the literature on misconception in mathematics

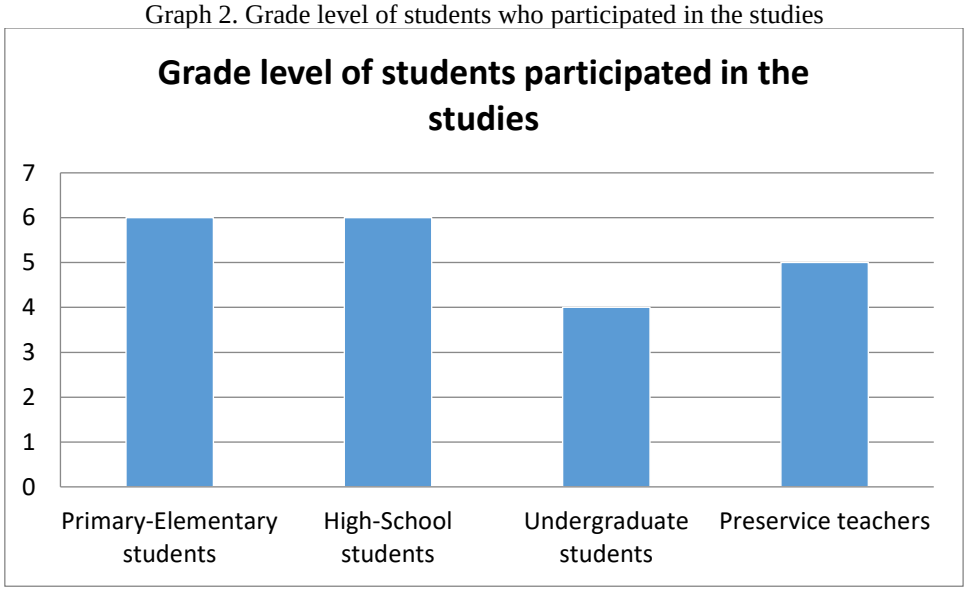
The articles examined were published between the years 2004 and 2015. According to Graph 1, it can be stated that there is an increase in the number of studies conducted on misconceptions in mathematics in recent years. We can obviously see that more than half of the articles have been published especially in the last five years.

Graph 1. Publication trends from 2004 to 2015



The studies were conducted on varied grade levels. According to Graph 2, the participants of six studies were primary and elementary students. Similarly, six studies were conducted with high school students. However, the participants of remaining studies (nine studies) were undergraduate students; in fact five of these were conducted with pre-service teachers.

Graph 2. Grade level of students who participated in the studies



The learning areas examined in the studies differed as well. As shown in Graph 3, the majority of the studies were related to *algebra* and *numbers* topics (n=3 for each). In addition, *derivative*, *trigonometry* and *limit* concepts were investigated in many of the studies (n=2 for each). Other learning areas investigated were *linear algebraic equations*, *arithmetic*, *inequalities*, *series*, *ratio-proportion*, *quadratic functions*, *fractions*, and *quadrilaterals* (n=1 for each).

The main purpose of the studies on misconceptions in mathematics: Eliminating or determining?

The studies were investigated in terms of whether they have just determined the types of misconceptions on a certain learning area or they have investigated the ways to eliminate these misconceptions. However, all studies except four studies had the purposes to determine what misconceptions students had on a specific mathematics subject, while only four studies applied an intervention in order to see whether the instructional technique or method was effective on eliminating the current misconceptions. For instance, Tuna's (2013) study compared the effect of the 5E model and traditional teaching methods on elimination of misconceptions about trigonometry concepts. The results demonstrated that the 5E model had an effect on non-emergence of misconceptions on trigonometry topic among students. In another study, Green, Piel, and Flowers (2008) investigated that manipulative-based instruction can efficiently and effectively help decrease the misconceptions in arithmetic and increase students' knowledge. Similarly, Gningue, Menil and Fuchs (2014) revealed that using virtual manipulatives helps students overcome misconceptions about algebra and pre-algebra concepts more easily. In the study of Koklu and Topcu (2012), it was found that the misconception scores and achievement scores of students who learned in Cabri-assissted learning environment were better compared with students' who were in traditional classroom.

Common data collection tools to determine misconceptions in mathematics

More than half of the studies included single data collection tool, while few of the studies were conducted using more than two tools. Among 22 studies, 15 studies used achievement or diagnostic tests. In these tests, 2 tests were designed as two-tiered (Koklu & Topcu, 2012; Lin, Yang, & Li, 2015) and 1 test was designed as four-tiered (Yang & Lin 2015). Researchers obtained data through lesson plans prepared by participants in 2 of the studies (Kula & Bukova-Güzel, 2014; Türnüklü, 2014). In addition, questionnaires were used to determine students' misconceptions in 3 studies (Almog & Ilany, 2015; Baştürk & Dönmez, 2011; Lin, Ko, & Kuo, 2014). Lastly, in 5 of the studies, interviews were conducted with participants in order to collect data (Almog & Ilany, 2015; Gningue, Menil, & Fuchs, 2014; Baştürk & Dönmez, 2011; Kaplan, İşleyen, & Dönmez, 2011; Kula & Bukova-Güzel, 2014).

DISCUSSION AND CONCLUSIONS

This systematic review of 21 articles on misconceptions in mathematics is conducted due to the lack of synthesis in the national and international context. Findings revealed the general approaches adopted in the studies in terms of participants, grade level, purpose, data collection tools and learning areas of mathematics which are investigated. First, the review found out that in recent years there is an increase in the number of studies about misconceptions on mathematics. Actually, this result is congruent with the findings of the study conducted by Türkdoğan, Güler, Bülbül and Danişman (2015), since the number of studies investigated in that study showed an increase in the last five years (between 2008 and 2013).

Second, most of the studies were conducted with primary, elementary and high school students, while less than half of the studies were conducted with undergraduate students. According to Duran (2013), during initial grades students learn many basic concepts in order to be aware of latter mathematics concepts, so researchers might have taken the road from this point of view and selected younger ages as participants.

This study revealed that articles investigated misconceptions on similar topics or learning areas of mathematics. While algebra and number sense are the most common subjects in primary and elementary classes, trigonometry, derivative and limit concepts are the most common topics in high school and higher education. In the related literature, although it is advocated that students have lots of misconceptions in many geometry topics (Dobbins, Gagnon & Ulrich, 2014), only misconceptions on quadrilaterals topic were investigated in one of the articles.

Most of the studies investigated in this review used traditional one-tier achievement tests. However, Peşman (2005) states that wrong answers given in these tests can be considered as misconceptions although they are not. Students can give wrong answer due to lack of knowledge, wrong information in the question or faulty thinking during the test, but these do not prove that student has misconceptions on that concept (Tunç, Akçam & Dökme, 2011). Therefore, tests that include more than two tiers are stated to be more appropriate to be able to detect the misconceptions (Peşman, 2005). However, it seems that very few studies took into consideration this point. In addition, one of the most appropriate ways to determine students' misconceptions is collecting qualitative data through interview or observation, since they provide in-depth information about students' knowledge. However, it is observed that only less than half of the studies used such methods in this review.

The results of this review proved that most of the studies were conducted for the purpose of determining students' misconceptions instead of eliminating them. Although determining is the first step of eliminating the

misconceptions, the researchers did not prefer going on further steps. Since people resist to the idea that their beliefs or thought are not correct, changing their concepts and eliminating misconceptions are not that easy (Comins, 1998). In addition, according to Sander (1993) it is too difficult to eliminate misconceptions through traditional learning and teaching techniques. Therefore, only few of the studies applied a treatment to see whether the treatment was able to eliminate students' present misconceptions.

Recommendations for Future Research

The major aim of the research on misconceptions in mathematics should move beyond just determining them. First of all, as educators we need more effective instructional methods which can prevent the arising of such misconceptions and eliminate the present misconceptions. Before this, it is also important to detect these misconceptions using correct tools. Therefore, researchers should be aware of which assessment techniques or data collection instruments are better at determining specific mathematics misconceptions.

This review investigates limited number of articles related to mathematics misconceptions. Thus, further review studies can be conducted to enlarge and overview more on misconception research. Especially in undergraduate level, there is a lack of studies related to mathematics misconceptions. In the related literature, teachers are stated as one of the reasons for misconceptions since they use inappropriate instructional methods or have lack of knowledge (Tekkaya, Çapa & Yılmaz, 2000). Hence, researchers can conduct more studies about in-service and pre-service teachers.

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