CHALLENGES OF 4th-YEAR MIDDLE-SCHOOL STUDENTS IN THE PROCESS OF MATHEMATICAL MODELING: SUMMER JOB PROBLEM

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ABSTRACT: The purpose of this qualitative study is to examine mathematical modeling processes of 4th year middle school students while working on a model eliciting activity, *the Summer Job Problem*, and to determine the difficulties encountered in the processes. This research was conducted in a middle school in a small county of a large city along the Black Sea Region of Turkey. Participants were 4th year middle school students in a state school. After a six-week preliminary study implemented on 24 students who had not experienced modeling before, the criterion sampling method was used to select three students that would be recruited into the focus group. The focus group was asked to work on the model eliciting activity of the *Summer Job Problem* and the entire process was recorded on video. A written transcript was made of the video recording, after which the recording and the students' worksheets were analyzed using the modeling cycle. The results of the study revealed that students expressed their ideas through discussions with students in the process, developed different assumptions and they appropriately could do mathematical calculations. On the other hand, students had difficulties (a) to interpret the data tables, (b) to identifying the variable of intensity, (c) use the main factors at the data tables and develop assumptions which includes these main factors, (d) justify the developed model.

Key words: Model Eliciting Activity, Mathematical Modeling, Summer Job Problem, Middle School Students

INTRODUCTION

The problems we face in the world become more complex considering the innovations and spread of technology. Especially educators in different disciplines emphasize that success only in school is not enough and it is important to train students who are able to overcome problem situations that include complex systems in order to achieve success beyond school (Lesh & Doerr, 2003). One of the things that can be used for this is mathematical modeling which includes model creation, analysis and correction (Lesh & Zawojewski, 2007; Romberg, Carpenter, & Kwako, 2005). NCTM's book "Principles and Standards for School Mathematics" (2000) emphasizes the need for the approach of modeling and that students should utilize mathematical models to show and understand numerical relationships from the pre-school period to the final year of high school. Mathematical modeling takes part in the school curricula starting from elementary school in various countries including Germany, the United States, Australia, the United Kingdom, Sweden and the Netherlands (Ng & Lee, 2015). Considering the updated middle school (5-8th grades) (MEB, 2013) mathematics curriculum, it may be seen that it encourages the improvement of the problem-solving capabilities of the students while promoting conceptual learning, being confident in operations and communicating with their mathematical knowledge. While focusing especially on problem-solving skills, it "approaches learning as an active process, emphasizes the need for students to take part in the learning process as active participants, and therefore projects that students will become subjects of their own learning process" (MEB, 2013). In this context, class environments where students can conduct investigations and questioning, establish communication, think critically, develop reasoning, share their opinions easily and present different solution methods should be established. In order to establish such learning environments, open-ended questions and activities that provide autonomy to students should be utilized and students should be provided with opportunities to study mathematics. The previous statement clearly describes how and in what kind of environment students are expected to solve problems. The curriculum emphasizes the need for the establishment of environments that make it possible for students to solve problems, develop different ways of representation of concepts and the relationships among these, discover mathematical relationships and improve their communication and reasoning skills. Therefore, in this study, modeling processes of 4th grade students were investigated with the help of model eliciting activities which include complex real-life situations, and the parts where the students experienced difficulties were determined.

When the national literature is reviewed, it is seen that the research on model eliciting activities is rare, but increasingly popular (Doruk and Umay, 2011; Delice and Kertil, 2015; Eraslan and Kant, 2015; Kal, 2011; Sandalcı, 2013; Şahin and Eraslan, 2016; Şahin and Eraslan, 2015; Şahin, 2014; Tekin-Dede and Bukova-Güzel, 2013; Tekin-Dede and Yılmaz, 2015). Due to the limited amount of research on the stages where students experience difficulties in the solution processes of model eliciting activities, the most general research question was determined as “what are the difficulties 4th-year middle school students face in processes of mathematical model eliciting?” in the light of the information in the literature, and the secondary issues were determined as the following in the data collection and analysis process:

1. Which thought processes 4th-year middle school students use during the activity of mathematical model eliciting?
2. Which difficulties 4th-year middle school students face in processes of mathematical model eliciting?

Theoretical Framework

Modeling, according to Lesh and Doerr (2003), is the act of organizing, coordinating, systematizing and ordering problem situations in the mind and finding a pattern, and eliciting models by using different schemes in the mind, in the process of interpreting (defining, explaining or establishing) events and problems. *Models* are conceptual systems that take part in equations, diagrams, software or other materialized representative media held in mind or applied in practice by students or problem-solvers (Lesh & Doerr, 2003).*Mathematical modeling* in this framework is the process of expressing a mathematical or non-mathematical real life situation in terms of mathematics, and it is a systematic process which includes various metacognitive activities such as analysis, synthesis and interpretation (Swetz & Hartzler, 1991). Lesh and Doerr (2003) defined mathematical modeling as a stage of model eliciting or a process that takes place during activities of model eliciting. Therefore, in the most general sense, *Model Eliciting Activities* are not general problems that are solved with a number or word at the end, but are problem situations that contain possible different solutions, which represent non-routine/complex real life situations, require individuals to mathematically interpret this situation, and describe or formulate the process of method mathematically, in order to help individuals who will benefit from this issue make decisions (Mousoulides, 2007; Lesh & Zawojewsky, 2007; Eraslan, 2011).

English and Lesh (2003) emphasized that the most important thing in modeling problems, as opposed to traditional problems, is not only reaching a goal, but the information that allows expression of the purpose and the possible steps of the solution. Lesh and Doerr (2003) stated that modeling problems are activities where students improve their research and discovery skills which contain transformation of real life problems into mathematics problems, planning how these problems are solved and ideas are developed, and making decisions on whether the ideas need revision or scope expansion and whether the ideas satisfy the conditions and assumptions given in the problem. Students spend most of their times developing various ways to think about relevant relationships, structures, systems and information. In this case, they actually change or transform their own characteristic ways of thinking regarding the data during the activity (Lesh & Doerr, 2003). In this study, in order to successfully explain students’ ways of thinking during model eliciting activities, OECD (2013) mathematization process and the modeling cycle by Blum (1996) were used together.

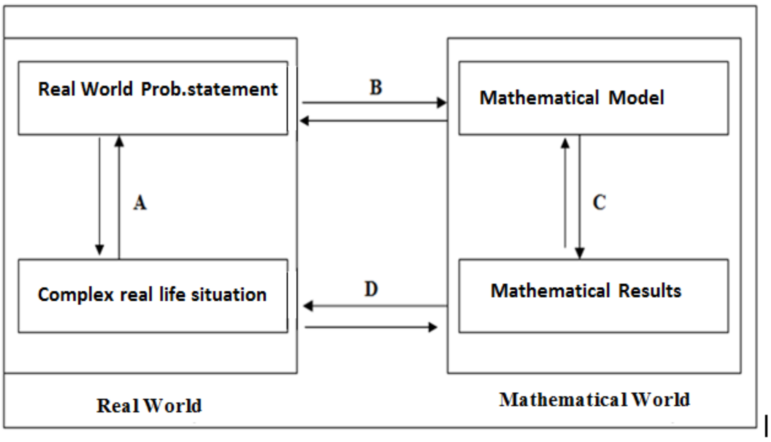

Figure 1. Modeling Cycle

They explained the cognitive processes that take place while transitioning between the steps on the modeling process above as the following: *A: In transition from the complex real life situation to the Real-world problem statement:* it is aimed to understand, simplify and make simplified assumptions about the *problem* and interpret the context. *B: In transition from the Real-world problem statement to the mathematical model:* team members are expected to determine the variables to be contained in the algebraic model, create a symbolic formula and form hypotheses that are related to the simplified assumptions they made in the process of understanding the problem. *C: In transition from the mathematical model to the mathematical results:* they are expected to apply the suitable symbolic formula and decide on the suitable mathematical operations, use mathematical tables for computation and obtain mathematical results that allow interpretation of the solutions. *D: In transition from the mathematical results to the complex real life situation:* it is aimed that the process comes back to the point it started, meaning, what is given and what is requested are compared, mathematical outputs are interpreted, the conclusion is approved and reported if the results are satisfactory, and the process is repeated if they are not.

METHOD

This is a qualitative study which aims to analyze model eliciting processes of 4th year middle school students, and determine and explain the reasons for the problems that may arise, if there are any. The study was designed as a case study, which is described as thoroughly investigating and analyzing a group or an event. The case approached in this study is to determine the mathematical modeling processes of three 4th year middle school students regarding the *Summer Job Problem.*

Study Group

This study was implemented at a state school with low socio-economic level in a small district of a metropolitan city in the Black Sea Region. The implementation was made at a 4th grade middle school class with twenty students. Firstly, before the main study, groups of three or four students were given a different model eliciting activity to work on every week for six weeks. While the researcher took an active role in this preliminary study as an implementer-teacher in the class who operated activities in person, the teacher of the class did not intervene. At the end of six weeks, the focus group of three students to be included in the main study was established using the *criterion sampling* technique, which is in scope of the method of *purposive sampling*. The following criteria were used to establish the group: the students were (a) able to work together in harmony during the six weeks, (b) talkative, self-confident and able to freely express thoughts, (c) academically successful.

Data Collection Tools

Following the six-week preliminary study which featured different model eliciting activities in the literature, the *Summer Job Problem* model eliciting activity was implemented with the group chosen from the class by the method of *purposive sampling*. The *Summer Job Problem* was adapted to Turkish from Johnson and Lesh’s (2003) study. The *Summer Job Problem* is a model eliciting activity that allows students to use their skills of interpreting mathematical and scientific information presented in the form of text and diagrams; reading data tables; analyzing and representing data; forming hypotheses on these, preparing a written report on the analyzed data; being able to work in groups and share the solutions reached at the end of the work (Johnson and Lesh, 2003). The focus group implementation which lasted a total of 90 minutes was video-taped; the data were sorted out, and analyzed qualitatively along the students’ work-sheets.

Data Analysis

The mathematical ideas developed and written responses presented by the 4th grade middle school students in the study during the solution of the *Summer Job Problem* were analyzed using the method of *descriptive analysis.* Descriptive analysis consists of the stages of: (a) establishing a framework for descriptive analysis, (b) processing the data based on the thematic framework, (c) describing the findings, and (d) interpreting the findings (Yıldırım and Şimşek, 2011). Therefore, the thought processes of the 4th grade middle school students in the focus group interview on model eliciting activities were analyzed using the modeling cycle adapted by the researchers (figure 1).

FINDINGS

The three students in the focus group were given the pseudonyms Sıla, Nur and Veli, while this section provides the discussion examples that reflect the difficulties they experienced in the processes.

A: Complex Real Life Situation $\longrightarrow$ Real World Problem Statement

While students are transitioning from the complex real life situation to the real-world problem statement, they are expected to understand the problem, simplify it, come up with simplified assumptions and interpret contexts. The following discussion reflects the difficulty the students faced in this stage:

Sıla: *is it the amount of things she sold, I do not get it*
Nur: *look, I found it, let me explain the table. Gizem, for example, sold this much in a day or a month or whatever when she was busy or earned this much money*
Sıla: *but look, Merve sold 34 of the goods when not busy and earned 765 TL of money*
Nur: *Sıla, these are not goods (products)*
Sıla: *Oh. How? But it says busy time. Where did it say it?*
Nur: *look, it is based on hours and months*
Sıla: *it says hours and months here, says very low average. I do not get if the hours fell behind*

According to the discussion above, Sıla thought of the working duration as the "*quantity of goods"* while Nur stated that it was not the quantity of goods, but number of hours. This shows that the students found it difficult to understand the working times table they were given. It is seen that the students were not able to thoroughly interpret the real-life situation and they could not "*understand*" that the hours in the table were the totals of the amounts workers spent working in different times.

B: Real World Problem Statement → Mathematical Model

While group members are transitioning from the real-world problem statement to the mathematical model, they are expected to determine the variables to be included in the algebraic model, create a symbolic formula and form hypotheses based on the simplified assumptions they made during the process of understanding the problems. The students formed the following hypotheses in this stage.

Veli*: now, for me, the time is not important. I think the ones earning the highest amount of money should be the best*
Nur*: but what is expected of us?*
Sıla: *but Veli, it is based on the time they work*
Veli*: we are going to find the best 3 employees*
Sıla: *Veli, they earn based on the time they work. So, one earns 300 million working for 1 hour, another earns 300 million working for half an hour. Which one?*
Veli*: okay, I think we will find both the one who brings and the one earns the highest amount of money*
Sıla: *we will also find the one who brings the highest amount of money in the shortest time.*

According to the text above, two different approaches were used in determining the main variable. The first hypothesis was Veli's "*time is not important, the one who brings the highest amount of money should be the best",* and the other one was Sıla's "*when it is low, the one who brings the highest amount of money in the shortest time should be the best.*" In this case, it is seen that the students used variables of *being busy* and *working time* interchangeably. It is seen that Sıla's proposal used the statements of *being less busy* and *short time* interchangeably. However, Sıla's approach was still more comprehensive than the first one as it contained both variables. Upon Sıla's proposal, they decided to calculate the *average monthly amount of money earned* as the following:

Sıla: *let us try to find it*
Nur*: we just had to understand the table*
Veli*: it seems the man did not want to do anything in August*
Nur*: now, I am saying it, what are you doing now**?***
Sıla: *let us do this part first. 474*
Nur*: 474 874 406 (Ali's earnings in June)*
Sıla *equals, divided by 3, equals 584. Let us not include the decimals, 'kuruş' (currency subunit) is not important. This is all June*
Nur*: there may be a problem later*
Sıla: *let us look at Ahmet. 1047 plus*
Sıla: *but we will find both the one who works full-time and the one who works part-time*
Nur*: and not everyone here has the same working hours*
Veli*: but, is it working hours?*
Nur*: yes, it is the hours worked*

The discussion above shows that while students did not "*understand the question"*, they tried to make sense of it through a set of mathematical operations. Sıla calculated the average money earned that she calculated for the worker *Gizem* in the month of June, for *Ali* this time. At this moment, they preferred to exclude the decimals of results and round them into integers. While the calculations made below contained the sub-variables of *June*, *July* and *August*, the students acted in compliance with Sıla's proposal to firstly find the averages of the data with *high, medium* and *low* density in steps, and then find the average of the three months.

C: Mathematical Model → Mathematical Results

In the stage of transition from the mathematical model to the mathematical results, students are expected to implement the suitable symbolic formula and decide on the suitable mathematical operations, use mathematical tables for calculation and obtain mathematical results that will allow interpretation of the solutions. The decision on the number of people they needed to choose as part-time and full-time and the calculations regarding these were discussed by the group members like the following:

> ***Veli***: *now, how many groups will this be?*
> ***Nur***: *three*
> ***Veli***: *seriously, three groups*
> ***Nur***: *not*
> ***Sıla:*** *we will divide it into three groups*
> ***Nur***: *but 6 people will work on the thing*
> ***Sıla:*** *3 people part-time, 3 people full-time, and the rest will be excluded*
> ***Veli:*** *no, three people part-time*
> ***Nur***: *look a little, three full-time, three part-time… oh, correct*
> ***Veli***: *did you get it, Nur?*
> ***Nur***: *we do not need three of them. No need for three*
> ***Sıla:*** *alright, we will divide them into three groups and dismiss the remaining three. They are the surplus. There is one left. I can make the histogram or you do it*

The students stated that they need to divide the workers into three groups by focusing on the presumption that they should choose the one with the highest earnings in the shortest time. These three groups were part-time workers, full-time workers and workers who would not work. As a result of discussions, while they agreed on using a *scoring system* to determine the workers, they faced difficulties in the necessary calculations. As a result, they combined the three months by taking the averages of the averages they previously found for the months of June, July and August. The students, who preferred to simplify their data by reducing the number of variables, determined the group gaps among the average earnings of the workers and their magnitudes. The students noticed that they had a miscalculation after they divided the workers into three groups, but they were only able to confirm the calculation of the average working time in three months. Later on, they tried to develop a scoring scheme to analyze both types of data (average earnings and working time). Their discussion at that moment was the following:

> ***Sıla:*** *look, now, one point each, one point. I will allocate the highest points to the ones in the first group*
> ***Veli***: *give highest points to these?*
> ***Sıla:*** *it will be three, two, one, and we will add these*
> ***Veli***: *why?*
> ***Nur***: *these are the ones who worked the least*
> ***Sıla:*** *there are 3 points, 2 points, 1 point. Let us add these. How many points does Ahmet have? 1 point from here, 2 points from here.*
> ***Veli***: *4*
> ***Sıla:*** *addition of two and one makes four?*
> ***Veli***: *three, three*
> ***Sıla:*** *Gizem has 1 point from here and 1 point from there. Gizem equals 2. Gizem has 2 points. Selim has 1 point here and 1 point there. Selim also has 2 points. Zeynep has 2 points from here and 3 points from here. Zeynep equals 5*
> ***Veli***: *this scoring system has good mathematics. I am glad I found a scoring system*

As seen from the discussion above, contrary to the presumption of "*choosing the one with the highest earnings in the shortest time"* that the students had agreed before, they gave three points to both the workers who worked the longest hours and the ones who earned the most, while reducing the points for others, resulting in a *scoring system* which also includes 2 points and 1 point.

D: Mathematical Results $\longrightarrow$ Complex Real Life Situation

Among the three groups established in this stage, while assigning scores to the groups in the working times table, the students disagreed the most on the group to assign the highest score (3 points). Sıla suggested assigning 3 points to the ones who worked the least, while Veli suggested assigning 3 points to the ones who worked the most; they discussed the issue as the following:

> ***Veli***: *just a second, look at this, there should not be confusion*
> ***Nur***: *the shortest time*
> ***Veli***: *just a second, why so? I am telling you, let us give higher scores to ones who work the most. They will work full-time. They should be the ones who work the most*
> ***Nur***: *but the one who makes the most in the shortest time makes more profit*
> ***Veli***: *but how will they work full-time? They should earn the most by working the longest*
> ***Nur***: *not the longest, but the shortest*
> ***Veli***: *but look, one will be part-time and the other will be full-time, is it not about that?*
> ***Nur***: *look, we are saying…*
> ***Veli***: *okay, go on*
> ***Nur***: *who would be more advantaged? Someone who writes the longest in the shortest time, or the longest time?*
> ***Sıla:*** *Veli, now there is Ayşe and Ahmet. One day, Ayşe…*

The quotes above show that the students discussed by presenting two different approaches while assigning scores to working times. While Sıla and Nur suggest assigning 3 points to ones who work for the "*shortest times*", Veli suggests assigning 3 points to the ones who work for the "*longest times*" as full-time workers should be the ones who work the most. However, later on, Veli is not insistent on his opinion and conforms to the decision of other group members. Then, the issue of creating different models for selecting part-time and full-time workers was discussed by the group members as the following:

> ***Veli***: *okay, okay. I get what you are saying, but I was talking about a different thing. I was talking about full-time*
> ***Sıla:*** *Veli was talking about full-time, but we cannot find part-time if we do this thing for full-time*
> ***Nur***: *but it becomes meaningless here, look, the method we found is different*
> ***Sıla:*** *so, if we do the full-time, part-time…*
> ***Veli***: *I get what you are saying, I get it. What I was saying was…*

The discussion above shows that the students thought they should sue different methods to determine the workers who would work full-time and part-time. However, it is seen that they accepted the previous assumption instead of creating a new model or symbolic formula. In other words, it was observed that the students saw the limitations and insufficiencies of the model they created by questioning it, but they did not create a new model by going back to the process of model eliciting. Later, they created a table by giving 3 point to the group with the least amount of working time. According to this table, the students determined that 5-point workers were full-time workers and 3-point workers were ones that would not work. However, they found it difficult to determine the third worker to work full-time, as well as workers who would work part-time. Later on, Veli gave the paper to the others and asked them to decide. Nur took the paper and decided on the workers as a result of the discussion with her peers:

> ***Veli***: *you find this last thing, I could not find it. I cannot comment*
> ***Nur***: *stop, stop. Let me take these 5-point ones for at least a minute*
> ***Veli***: *never mind it, find this other side*
> ***Nur***: *the one you and I made will be different. Is Fatih not the last one? Fatih, the 4-point ones… Where is Selim? This will be the last one. Who is before Selim? Gizem. This one also gone*
> ***Veli***: *go down starting with high scores*
> ***Nur***: *keep quiet a little, Veli. You should have done it, then. Gizem, Ahmet. This will come before, and this one after. Then other 4-point ones…*
> ***Nur***: *Selim. Why did you put down Ayhan? Ayhan does not have 4 points. Why did you write his name to the top? Look, it is the last one from the start. You got it right. Ayhan, Selim, Gizem. It is done*

The quotes above show that Nur determined the workers who would work only by looking at the tables, without developing any strategy. In other words, it is seen that the students were not able to interpret the situation they encountered by going back to the complex real life situation while deciding on workers with the same score, improve their models by forming new assumptions, or create alternative models. The group members approved the result without any discussions after determining the workers as in the process above, and completed this activity by creating a written report.

CONCLUSION

The results obtained from the groups during the model eliciting process show that 4th grade middle school students face some difficulties in the process of mathematical model eliciting. In the *transition stage from the complex real life situation to the real-world problem statement,* the students found it difficult to make sense of the variables in the problem and tried to understand the problem over one variable by simplifying the data. In this stage, the students experienced difficulties in the stage of understanding the problems as reported in the studies Blum and Leiβ (2007) and Sol et al. (2011). The students especially had difficulties in making sense of the concept of being busy and the relationship between the money earned and the hours worked. In order to overcome this difficulty, the students tried to understand the problem by repeatedly re-reading the problem and the data table. A set of mathematical relationships were sought among the data and simplified assumptions were formed regarding the criteria to be analyzed collectively.

In the next stage of *transitioning from the real-world problem statement to the mathematical model,* the students had difficulty in developing a model by approaching all variables comprehensively and determine the main variable. As also reported in the studies of Blum and Leiβ (2007) and Schapp et al. (2011), this situation shows that the students were not able to use or relate all variables. Hypotheses were related to real life and the ones that were not found relevant were not accepted as true. As in Kaiser's (2007) and Blum and Leiβ's (2007) studies, the students experienced difficulties in establishing a suitable model and structuring that model.

In the stage of *transitioning from the mathematical model to the mathematical results,* the group members made various mathematical calculations on the models they created. They noticed the operation mistakes they made, related their results to real life, and dismissed their operations and hypotheses in cases where those were found unrealistic. The students found it difficult to use the given variables together, and they obtained mathematical solutions which represented the real-life situation by mathematical calculation towards discovering the relationships among the components of the given variables. Because they experienced difficulty in using the data collectively, they determined a scoring scheme as an analysis criterion by simplifying the data. The students, regarding the difficulty in grouping the workers based on this criterion, developed an unsystematic solution and completed the process. This may be caused by the students' desire to reach a solution quickly, without spending a sufficient amount of time to understand and analyze the problem (Blum and Ferri, 2009).

In the stage of *transition from the mathematical results to the complex real life situation,* the group members did not choose to check or question whether the established groups are correct or not. This situation is in parallel with Kaiser's (2007) and Galbraith and Stillman's (2006) results that students find it difficult to interpret the mathematical results they obtained for real life. The group members chose to accept the solution as it is, without reviewing the model. This approach shows that, in similarity to Maaβ's (2007) study, the students were not aware that the model's validity needed to be established.

Consequently, during the modeling process, the group members experienced difficulties in understanding the problem to develop models suitable for the real-life situation, discovering the relationships among the components of the qualitative variable, relating all variables to each other, forming hypotheses suitable for the data, creating the suitable model based on these hypotheses, and establishing a connection between real life and mathematics by establishing the validity of the model. One reason for this may be the effects on the modeling process by personal characteristics of the group members such as habits based on previous experiences that shape their ways of mathematical thinking, attitudes towards mathematics, and creative thinking skills (Chamberlin, 2004; Ferri, 2011). Additionally, students' expectations from the activity and whether they find the activity interesting or not, are shown among the factors that affect this process (Schoenfeld, 1992). When the work of the students during modeling activities was analyzed, it was observed that mathematical modeling activities have a very strong aspect which contributes to students' communication skills and provides opportunities for them to improve these aspects of theirs. When modeling activities are implemented in the form of team work, students show behaviors in the form of asking critical questions, expressing, defending and proving opinions, and persuading their peers (Zawojewski, Lesh and English, 2003). Additionally, when students express their ideas during the solution of the problem, they actually take part in a different activity which gradually assesses their own ways of thinking.

RECOMMENDATIONS

It is seen that a modeling approach is adapted for the *Mathematical Applications* course put in place in 2012, and similarly structured activities are covered to achieve the modeling activities of the course. However, it is seen in the results of the conducted research that these activities and the course itself fall short of the expectations. This situation shows the necessity of in-service training for teachers who are the implementers of this program towards the purposes, outcomes and necessity of model eliciting activities. Moreover, inclusion of mathematical modeling skills among basic skills in the primary school (1-4th grades) mathematics curriculum necessitates the expansion of mathematical modeling classes in teacher training programs to also include primary school teachers.

REFERENCES

Blum, W. (1996). Anwendungsbezüge im Mathematikunterricht–Trends und Perspektiven. *Schriftenreihe Didaktik der Mathematik*, 23, 15–38.

Blum, W., & Leiß, D. (2007). How Do Students and Teachers Deal With Modeling Problems? In C. R. Haines, P. Galbraith, W. Blum & S. Khan (Eds.), *Mathematical Modeling (ICTMA–12): Education, Engineering and Economics* (pp. 222–231). Chichester: Horwood Publishing.

Blum, W., & Ferri, B. R. (2009). Mathematical Modeling: Can It Be Taught and Learnt? *Journal of Mathematical Modeling and Aplications*, 1(1), 45–58.

Chamberlin, M. T. (2004). Design Principles for Teacher Investigations of Student Work. *Mathematics Teacher Education and Development*, 6, 61–72.

Delice, A., & Kertil, M. (2015). Investigating the Representational Fluency of Pre-Service Mathematics Teachers in a Modelling Process.*International Journal of Science And Mathematics Education*, 13(3), 631-656.

Doruk, B. K., & Umay, A. (2011). The Effects of Mathematical Modeling on Transferring Mathematics into Daily Life, *Hacettepe University Journal of Education*, 41, 124-125.

English, L. D., & Lesh, R. A. (2003). Ends in-view Problems. In R. A Lesh ve H. M. Doerr (Eds.), *Beyond Constructivism: A Models and Modeling Perspective on Mathematics Problem Solving, Learning and Teaching* (pp. 297–316). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Eraslan, A., & Kant, S. (2015). Modeling Processes of 4th-Year Middle-School Students and the Difficulties Encountered. *Educational Sciences: Theory & Practice*, 15(3), 809-824.

Eraslan, A**. (**2011**).** İlköğretim Matematik Öğretmen Adaylarının Model Oluşturma Etkinlikleri ve Bunların Matematik Öğrenimine Etkisi Hakkındaki Görüşleri. *İlköğretim Online,* 10 (1), 364-377.

Ferri, B. R. (2011). Effective mathematical modelling without blockages- a commentary. In G. Kaiser, W. Blum, R. B. Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling: The 14. ICMTA study* (pp. 181–185). New York, NY: Springer.

Galbraith, P., & Stillman, G. (2006). A framework for identifying student blockage during transitions in the modelling process. *Zentralblatt für Didaktik der Mathematik*, 38(2), 143–162.

Johnson, T., & Lesh, R. A. (2003). A Models and Modelling Perspective on Technology-Based Representational Media. In R. Lesh & H. M. Doerr (Eds.), *Beyond Constructivism: A Models and Modeling Perspective on Mathematics Problem Solving, Learning and Teaching* (pp. 265–278). Mahwah, NJ: Lawrence Erlbaum Associates.

Kaiser, G. (2007). Modelling and modelling competencies. In C. R. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling, education, engineering and economics: The ICTMA 12 Study* (pp. 10–119). Chichester, UK: Horwood Publishing.

Kal, F. M. (2013). *Matematiksel Modelleme Etkinliklerinin İlköğretim 6.Sınıf Öğrencilerinin Matematik Problemi Çözme Tutumlarına Etkisi*. Yayınlanmamış Yüksek Lisans Tezi, Kocaeli Üniversitesi, Fen Bilimleri Enstitüsü.

Lesh, R. A., & Doerr, H. (2003). Foundations of a Models and Modeling Perspective on Mathematics Teaching and Learning. In R. A. Lesh & H. Doerr (Eds.), *Beyond constructivism: A models and modeling perspective on mathematics teaching, learning, and problem solving* (pp. 3–34). Mahwah, NJ: Lawrence Erlbaum and Associates.

Lesh, R. A., & Zawojewski, J. S. (2007). Problem solving and modeling. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 763– 804). Charlotte, NC: Information Age Publishing.

Maaß, K. (2007). Modelling Taks for Low Achieving Students.First Results of an Empirical Study. In D. Pitta-Pantazi & G. Philippou (Eds.), *CERME 5 – Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 2120–2129). Larnaca: University of Cyprus.

MEB (Milli Eğitim Bakanlığı). (2015). *İlkokul Matematik Dersi (1–4) Öğretim Programı*. Ankara: Talim ve Terbiye Kurulu Başkanlığı.

MEB (Milli Eğitim Bakanlığı). (2013). *Ortaokul matematik dersi (5, 6, 7 ve 8. Sınıflar) öğretim programı*. Ankara: Talim ve Terbiye Kurulu Başkanlığı.

MEB (Milli Eğitim Bakanlığı). (2013a). *Ortaokul ve İmam Hatip Ortaokulu Matematik Uygulamaları Dersi (5, 6, 7 ve 8. Sınıflar) Öğretim Programı*. Ankara: Talim ve Terbiye Kurulu Başkanlığı.

Mousoulides, N. (2007). *A modeling perspective in the teaching and learning of mathematical problem solving* (Doctoral dissertation, University of Cyprus, Cyprus). Retrieved from on 15- November 2015, ar URL: http://lekythos.library.ucy.ac.cy/handle/10797/5927

Ng, K. E. D., & Lee, N.G. (2015). Introduction: Mathematical Modelling Outreach in Singapore In: Lee, N.G. & Ng, K. E. D (Eds.), *Mathematical modelling: from theory to practice* (pp. 1-19). National Institute of Education, Nanyang Technological University, Singapore.

OECD (2013), "PISA 2012 Assessment and Analytical Framework: Mathematics, Reading, Science, Problem Solving and Financial Literacy". [Online] Retrieved on 10-June- 2015, at URL: http://www.oecd-ilibrary.org/education/pisa-2012-assessment-and-analytical-framework_9789264190511-en

Romberg, T. A., Carpenter, T. P., & Kwako, J. (2005). Standardsbased reform and teaching for understanding. In T. A. Romberg, T. P. Carpenter & F. Dremock (Eds.), *Understanding mathematics and science matters*. Mahwah: Lawrence Erlbaum.

Sandalcı, Y. (2013). *Matematiksel Modelleme İle Cebir Öğretiminin Öğrencilerin Akademik Başarılarına ve Matematiği Günlük Yaşamla İlişkilendirmelerine Etkisi*. Yüksek Lisans Tezi, Recep Tayyip Erdoğan Üniversitesi, Sosyal Bilimler Enstitüsü.

Schapp, S., Vos, P., & Goedhart, M. (2011). Students overcoming blockages while building a mathematical model: Exploring a framework. In G. Kaiser, W. Blum, R. B. Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling: The 14. ICMTA Study* (pp. 137–146). New York, NY: Springer.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition and sense making in mathematics. In D. Grows (Ed.), *Handbook of research on mathematics teaching and learning (pp. 334-370). New York*, NY: Macmillan.

Sol, M., Giménez, J., & Rosich, N. (2011). Project Modelling Routes in 12–16-Year-Old Pupils. In G. Kaiser, W. Blum, R. B. Ferri, G. Stillman (Eds.), *Trends in Teaching and Learning of Mathematical Modellin: The 14. ICMTA Study* (pp. 231–240). New York: Springer.

Swetz, F., & Hartzler, J. S. (1991). *Mathematical modeling in the secondary school curriculum*. Reston, VA: NCTM.

Şahin, N., & Eraslan, A. (2016). Modeling Processes of Primary School Students: The Crime Problem. *Education & Science*, 41 (183), 47-67.

Şahin, N. & Eraslan, A. (2015). *Middle School Students' Modeling Procesess: Summer Reading Problem*. International Conference on Education in Mathematics, Science & Technology (ICEMST). 23- 26 April, 2015, Antalya, Turkey.

Şahin, N. (2014). *İlkokul4.Sınıf Öğrencilerinin Model Oluşturma Etkinlikleri Üzerindeki Düşünme Süreçleri*.Yüksek Lisans Tezi, Ondokuz Mayıs Üniversitesi, Eğitim Bilimleri Enstitüsü.

Tekin-Dede, A., & Bukova-Güzel, E. (2013). Examining the Mathematics Teachers' Design Process of the Model Eliciting Activity: Obesity Problem.*Elemantary Education Online*, 12(4), 1100-1119.

Tekin-Dede, A. ve Yılmaz, S. (2015). 6. Sınıf öğrencilerinin modelleme yeterlilikleri nasıl geliştirilebilir? International Journal of New Trends in Arts, Sports&ScienceEducation, 4(1), 49-63, [online]: http://www.ijtase.net/ojs/index.php/IJTASE/article/viewFile/365/458 adresinden 30 Aralık 2015 tarihinde indirilmiştir.

Yıldırım, A. ve Şimşek, H. (2011). *Sosyal Bilimlerde Nitel Araştırma Yöntemleri*. Ankara: Seçkin Yayıncılık.

Zawojevski, S. J., Lesh, R. A., & English, L. D. (2003). A models and modeling perspective on the role of small group learning activities. In R. A. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: A models and modeling perspective on mathematics problem solving, learning and teaching* (pp. 337– 358). Mahwah, NJ: Lawrence Erlbaum Associates.