Teaching through Research: Proposal of a Didactic Device for High School

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Introduction

One of the main problems that mathematics education faces is the general reluctance citizens feel. In particular, this is due to the fact that school has forced generations of students to visit mathematical notions that lack reason to be (Chevallard, 2017). In the last Anthropological Theory of the Didactic (Chevallard, 2013a, 2013b, 2017) a new paradigm for the study of mathematics is advocated. This is known as world questions paradigm, which is based on the principle that education is a life-long process and the approach to knowledge is intrinsically motivated by the study need. It is a functional study justified by the problem to be solved. The community studies a question where the inquiry is deepened, leading students to find or re-find works to build an appropriate answer.

In this work, a didactic device for high school is proposed. The goal is to discuss, explore, conjecture, ask, create new problems and promote a genuine mathematic activity. The proposal adopts an epistemological conception that considers mathematics as a functional and useful knowledge. The didactic device starts with a question that demands to compare the cell companies' plans. It implies to analyze an everyday situation any citizen may experience, where neither the data nor the variables are fully determined beforehand. The situation is inspired in the proposal of Rodríguez, Hidalgo, Sierra (2013), who suggests comparing and analyzing situations close to reality. However, my proposal requires students to compare the fees of cell telephony analyzing the current state of companies in Argentine.

In relationship with the epistemological conception of the Anthropological Theory of the Didactical (ATD), this situation generates a mathematical activity which is a bit unusual in the current teaching system. This activity consists in formulating and answering questions, searching in different media, developing different techniques, making conjectures, validating solutions, interacting with other members of the group while comparing results, techniques, validations, etc. In this proposal it is fundamental to break with the atomised conception of the mathematics and to go over different mathematical organizations according to study needs.

The role of the traditional teacher needs to change. He has to move from the habitual place accepted in the institution, and abandon the function of protagonist to give way to the students doing. The teacher has to be the director of the study and research

process, able to influence opportunely and effectively to foster the study. The teacher´s study help must enable the students to find where the problematic of the situation is as well as to keep the study meaningful.

Framework

The Anthropological Theory of the Didactic (Chevallard, 1999, 2007, 2013a, 2013b, 2017) proposes to introduce functional study processes in the teaching system. It aims to analyze fundamental questions (in general non-mathematics), which give rise to the curriculum. The study of these questions generates new questions to study, and reveals the necessary work to use. In this paradigm, the leadership of the questions takes precedence (and the corresponding answers to them) attenuating the leadership of the work that traditional pedagogy allocates. This proposal is materialised in a didactic device named Study and Research Paths (SRP). This device supports a new scholar epistemology based on the research and the world question. The methodology of the new paradigm needs to add didactic gestures, which involve changes with respect to traditional teaching. This proposal would permit to confront the didactic phenomena identified by Chevallard (2013a) as monumentalisation of the knowledge, typical of the visit work paradigm. Here, the study is found directly and its study is motivated by a formal decision according to traditional lineal organisation.

A typical didactic system of the research paradigm is characterized by a student group *X* that study a questions Q with the help of a teacher or teacher group *Y*, who together contribute a possible answer to . The production of requires that the didactic system S has instruments, resources, works. That is, the system needs to generate a didactic medium *M* to produce .

The SRP begins with a question suggested by the teacher. The study leads to find or re-find several mathematical organisations and other disciplines. In this way, it follows that the question-answer pairs are the core of the study processes . Here are all the questions in the core ♥ and are the corresponding answers for each (Chevallard, 2007).

The didactic gestures typical of the study and research named dialectics are required to implement a SRP (Chevallard, 2007, 2013b). Next, five essential dialectics can emerge from processing the situation study suggested in this paper:

- *The study and research dialectic*. A research supposes the study of the combination of questions and pre-established answers. This dialectic is the engine of teaching for SRP. We cannot research without studying. In addition, a genuine study is the generator of questions to be researched.

- *The media and middle dialectic.* The productions of provisional successive answers

require pre-established answers, accessible by means of communication and diffusion: the media (book, paper, class note, etc.). These answers are the product of conjecture, therefore they must be checked before being transformed and incorporated to the medium.

- *The individual and group dialectic.* The students with the study director must distribute the task and negotiate the responsibility.

- *The dialectic of getting in and out of topic*. If the question is open and brainstorming, it is likeable to get out of the topic, even it could be necessary to get out of the reference discipline and get back in later.

- *The dialectic dark boxes and the light boxes.* This refers to the need to establish whether a work deserves to be studied, clarified analyzed, etc., or some knowledge is approached at grey level. Here is not essential to answer the brainstorming questions and its derivate questions.

A teaching by SRP presupposes the formulation of questions and its study in agreement with the full group. This demands to share out responsibility and allocate individual task. Thus, the group process of producing answers is resumed. The works found or re-found to produce the answers will be studied with certain level of depth to establish its pertinence. Moreover, during the study new questions can appear, which the study community will decide when and how to answer. Therefore, the responsibility of the study does not fall in the individual but rather in the producing community which holds and validates the answers generated collectively.

Next, the description of a situation is suggested. Its study allows that some typical gestures of the pedagogy of research emerge.

Description of the didactic proposal

This didactic device starts in situation 1. The analysis of the situation generates questions not planned from the start. Here a possible path is proposed. The activity can be developed in several directions. Studying the questions with more or less depth will depend on the students'general interest.

The teacher´s activity consists in supervising the productions of the students and regulating the time to systematize the discussion of the different groups. It is fundamental that the students keep the initial situation meaningful. They should avoid giving immediate answers because they impede to problematize and find the functionality of mathematics to provide an answer.

First, the initial situation is presented:

Situation 1

There are several Cell Companies in the market. We want to contract one. Which one? Why?

Next, possible paths that can emerge during the study are described. In addition, situations are proposed to develop the study as a result of either the students´ proposal or the teacher´s formulation.

In this way, it is essential that learners group come up with the need to play formulating questions and answers which do not provide an immediate answer.

As a first step, the teacher can ask the learners: "What cell companies do you use?" "Why did you choose these cell companies?" The students are expected to formulate new questions focused on the analysis of the plans of the cell telephony. This information is available on the web pages of each cell company but, as it is varied, students need to organize it to compare and decide how to study it.

Currently, three companies in Argentine offer the following information about their plans:

- Fee plan charge
- Megas available to use internet.
- Telephone number to talk unlimitedly.
- Available credit to all consumption.
- SMS included in the plan
- Purchase cost to use internet once the available credit is over.
- Form and cost of local and long distance calls.
- Form and cost of local and long distance calls once the available credit is over.
- Access to music and video.
- Available minutes to talk included in the fee plan charge.

Analyzing the fee plan charge we can understand how the cell companies value the communication by telephonic call. The resolution Nº 26/2013 of the Ministry of federal planning, public investment and services in Argentine, establishes that the first 30 seconds of the communication have a fixed cost, then the cost is valued per second.

Next, the situation 1 is developed. This leads to go through different mathematics praxeologies, some of which form part of the curricula of the high school in Argentine.

The exploration of situation 1 generates new questions. Firstly, if we want to contract one cell company, we have to establish our needs. Therefore, we must study the following questions:

Situation 2

How do we use our cell?

For each client, this information is available in cell telephone or in personal profile of the web page company.

To establish what we use our cell for, we to need collect and synthetize information about contact number, register call, SMS use, etc. Thus, the student will be able to study fundamental tasks about descriptive statistics, such as: collect data, organize data in tables, analyze values of categorical variables and numerical variables, summarize and represent data, communicate information from statistical studies. This task leads, for example, to establish temporal space to register the data and organize them in tables. In this organisation basic notions of statistics such as relative and absolute frequency may participate.

The study depends on the personal use that each student makes of his cell phone. For example, the proliferation of the use of WhatsApp to establish communication via message generates that SMS turn out obsolete or only employed by the company of cell telephony for publicity. If the study does not progress and the questions formulated do not suggest further depth, I propose the study of the next situation.

Situation 3

What other forms of communication can we establish with the cell?

In relation to the study of situation 3, it is possible to increase the information about the cell phone use beyond the personal scope. To start this analysis, the information should result useful to make a survey. This is proposed in situation 4:

Situation 4

Make a survey to gather information about how secondary students use their cell phones.

These situations lead us to the problematic: How is a survey prepared? How do we implement the survey? How is the generated data analyzed? How are the results reported? Again, the study of data from descriptive statistic is vital. The analysis also involves the use of some software to summarise data volume.

If we go back to the initial situation, we can also study the following:

Situation 5

What data are considered to compare the fee of cell telephony?

To compare the fee of cell telephony, it is necessary to consider the aspects indicated in the description of the situation 1.

The megas to use internet can be quantified from different applicative to cell telephone. This is complex to analyze because it depends on the applicative chosen by the user. For the final analysis the megas offered by each company will be considered in relation with the plan cost.

The access to music and video depends on each company. Some companies offer this package at zero cost for a limited period.

The communication by SMS is simple to analyze. The SMS cost is given for unit cost. If we want to know which company offers the lowest SMS cost, we can analyze the companies' information and decide. To calculate the SMS cost we can use the following expression:

$$m(x)=kx$$

The expression m(x) belongs to a direct proportionality function. Here, represents the message number and the unit cost of the message.

In an ideal case, all available credit for all consumption (c_r) included in the payment is used to send SMS. In this case, [c_r/k] SMS can be sent as maximum. In this expression, we used entire part function because if the quotient does not turn out to be a natural number, the resultant decimal of the operation is not sufficient to cover the send cost of a message.

Finally, the domain function m(x) is:

$$dom\big(m(x)\big) = \left\{x \in \mathbb{N} / 1 \leq x \leq \left[\frac{c_r}{k}\right]\right\}$$

To decide which company has less cost to send SMS is sufficient to compare the pending of m(x) for each company. Also, this should be studied in relation with the available credit to all consumption of each plan. The company offers different cost to send SMS and available credit to all consumption. Sometimes, the company that offers the lowest cost to send SMS does not always offer the most available credit to all consumption. Thus, the study must be made in a comprehensive way.

The most problematic of the situation is when we want to analyze the telephonic call cost. So, we have to understand how the company rates the telephonic call. In this

regards, the following situation is propose to study.

Situation 6

How to calculate the cost of a call?

According to resolution Nº 26/2013 of the Ministry of federal planning, public investment and services of the Argentine Republic, the way the call is valued by companies of cell telephone is a fixed cost to the first 30 seconds once established communication and, then the cost is valued per second.

Regarding the plans that the companies offer, the students might give immediate answer, indicating the available information on the web page of each company. This situation is not problematic as the students do not find the usefulness of maths to answer. In this situation, I propose to study the following:

Situation 7

What is the cost of all the calls registered in the personal cell telephone? How to calculate it?

Situation 8

What is more advisable: talk t seconds in one call or in several calls? Why?

Given available information in the plans that the telephone cell companies offer, it follows that to calculate the cost of a call, the companies will invoice a fixed cost that considers the first seconds of communication. Then, the companies establish a cost for each extra second that the communication goes on.

Finally, the group can obtain the next mathematical model to calculate the cost of a t seconds call.

$$c(t) = \begin{cases} c_e & si \quad 1 \leq t \leq t_e \\ c_e + c_s(t - t_e) & si \quad t_e + 1 \leq t \leq \left\lfloor \frac{c_r - c_e}{c_s} + t_e \right\rfloor \end{cases}$$

Where t_e is the maximum time that contemplates the cost for the establishment of the call. c_r is the credit for all consumption. c_e is the cost for establishing the call. Finally, c_s is the cost for each second after the t_e seconds once established the communication.

The expression cost c(t) belongs to a function defined by pieces. This is composed by a branch that belongs to the expression of a constant function and another one that belongs to the expression lineal function. The latter is positive and increasing because $c_e>0$ and $c_s>0$. In addition, c_s t_e is smaller than c_e , otherwise the situations would not have sense. Were c_s $t_e \geq c_e$, to some values of , the cost of the call would be negative. Namely, it would indicate loss to the companies and balance in favour of the user. In this

way, the minimum cost of a call is c_e.

The domain of is:

$$Dom(c(t)) = \left\{t \in \mathbb{N} / 1 \leq t \leq \left\lfloor \frac{c_r - c_e}{c_s} + t_e \right\rfloor\right\}$$

t represents the time which is continuous. However, in the problem context *t* is considered to take natural values by the form of call valuation of the cell telephone companies.

The companies value the time a communication lasts by the roof function. For example, if a telephone communication lasts 41.6 seconds, the companies will invoice 42 seconds. However, if the time *t* is considered as continuum ($t \in \mathbb{R}^+$) the second function branch c(t) needs transforming in $c_e + c_s([t] - t_e)$. In this paper we will consider that because the registration of the last call in the cell phone or on the personal web page is in seconds instead of split seconds.

Next, the function c(*t*) is analyzed. The branch that involves a constant function represents the cost of a call which lasts from 1 to t_e seconds. In this term, all the calls have the same cost and are worth t_e seconds. In turn, to calls which last over t_e seconds, the cost function corresponds to a lineal function. In this case, the validity needs to be analyzed.

The telephone cell companies stipulate an available credit to all consumption (c_r) for each plan they offer. Suppose that c_r were used in a continuous call to maximize its use. If we consumed c_r in telephone communications of different duration, the cost of the call would always be affected by the cost of the communication establishment.

Thus, we may be able to maintain a communication that lasts at most $\left\lfloor \frac{c_r - c_e}{c_s} + t_e \right\rfloor$ seconds with available credit to all consumption that the companies stipulate, the maximum time to talk is the result of the following equation:

$$c_e + c_s(t - t_e) = c_r$$
$$t = \frac{c_r - c_e}{c_s} + t_e$$

The floor function is used in the expression $\left\lfloor \frac{c_r - c_e}{c_s} + t_e \right\rfloor$, because according to the value to each parameter, the result of the operation can be a rational number. The split seconds that are the result of the calculus are lost by the user. The companies fix the communication per second duration.

To deepen the analysis of the comparison among cell companies is interesting to analyze current situations. The three companies hereto considered propose the following plans. This data correspond to the most economical plan of the three companies dated

October 2017 in Argentine:

Company	Plan cost	Internet included	Available credit to all consumption	SMS Included	SMS Excess	Call connection (First30 seconds)	Second included cost	Minutes free to call other companies
Company 1	$250	1G	$55	100	$1,80	$1,5780	$0,0526	-----
Company 2	$299	2G	$30	Unlimited	$ 2,67	$2,9251	$0,0975	200
Company 3	$260	1G	$30	Unlimited	--------	$2,31	$0,077	200

Suppose we wanted to know how long we can call with the available credit full to consumption credit offered by each company. In Company 1 the plan allows to call 1045 seconds (approximately 17 minutes). In Company 2, the plan allows to call 307 seconds (approximately 5 minutes). In Company 3 to call 389 seconds (approximately 6 minutes).

In particular, the Company 1 plan offers lower cost of both the call connection and seconds than the plan of the Companies 2 and 3. Also, the full credit consumption is higher in the plan offered by Company 1. Finally, we conclude that it is possible to have calls of similar duration for the plans offered by the Companies 2 and 3.

If we study totality plan, the companies 2 and 3 included 200 minutes to call other companies and they offer the same full credit consumptions. I conclude that it is possible to have calls of similar duration for both plans. It is only for communications nearer than 30 kilometers though, as the Company 2 applies additional cost.

The analysis of call cost can be deepened studying the functions of the particular case of each company. However, it is suggested moving away from particular situations to study possible cases we might face. In this sense, the following situation is proposed:

Situation 9

Considering two cell companies that offer different cost for call connection and the cost per second after seconds once connected, which company is more economical in relation with the duration of the call?

Suppose that cost function for two cell companies were the following.

$$c_1(t) = \begin{cases} c_{e_1} & si\ 1 \leq t \leq t_e \\ c_{s_1}t + c_{e_1} - c_{s_1}t_e & si\ \ t_e + 1 \leq t \leq \left\lfloor \dfrac{c_{r1} - c_{e_1}}{c_{s_1}} + t_e \right\rfloor \end{cases}$$

$$c_2(t) = \begin{cases} c_{e_2} & si\ \ 1 \leq t \leq t_e \\ c_{s_2}t + c_{e_2} - c_{s_2}t_e & si\ \ t_e + 1 \leq t \leq \left\lfloor \dfrac{c_{r2} - c_{e_2}}{c_{s_2}} + t_e \right\rfloor \end{cases}$$

In both expressions, the expression of the second branch involving a lineal function was modified by means of algebraic techniques. In particular, we aim at obtaining an

expression as f(x)=ax+b, thus leaning the composition of parameter *b*. This depends on c_e and c_s. It results that $b=c_e-c_s t_e$. *b* is a positive parameter because c_e,c_s, t_e are fixed cost and $c_e \geq c_s t_e$.

The study of the situation 9 analyzes to what time values *t*, result $c_1(t)=c_2(t)$, $c_1(t)<c_2(t)$ y $c_1(t)>c_2(t)$. To compare it is essential to interpret the involved situation. The particular cases of each company may be studied by arithmetic, graphic and algebraic techniques. Here only the algebraic techniques are used as they justified to others.

To compare the call cost of each company we need to analyze the possible values of the parameters that make up the expression (c_e and c_s) will not be analyzed here because it is a set value and identical to all companies. This allows to predict possible cases:

Case 1: If $c_{e_1} = c_{e_2}$ and $c_{s_1} < c_{s_2}$ or if $c_{e_1} = c_{e_2}$ and $c_{s_1} > c_{s_2}$

For both companies, the call connection cost is the same, but it is higher at t_e seconds in one of them.

The second branch in the cost function c(*t*) between both companies differ in origin ordinate and slope. Therefore, a period in which the cost for both companies will be the same can be established. This depends on the value of cost per call connection (c_e), the cost of the second after seconds of the call connection, and the available credit to full consumption (c_r). After this period, the company with higher cost of the second after t_e seconds of call connection will be the most expensive company. To know the time where both companies have the same cost we propose:

$$c_{e_1} + c_{s_1}(t_i - t_e) = c_{e_2} + c_{s_2}(t_i - t_e)$$

For $t_i = \frac{c_{e_2} - c_{e_1}}{c_{s_1} - c_{s_2}} + t_e$ the cost of the communication is the same for both companies. The moment t_i equals t_e. Since $c_{e1} = c_{e2}$, emerges that $t_i = 0 + t_e$, e. i. $t_i = t_e$. For communications that last 1 a t_e seconds, the cost is the same for both companies.

Case 2: If $c_{e_1} > c_{e_2}$ and $c_{s_1} < c_{s_2}$ or if $c_{e_1} < c_{e_2}$ and $c_{s_1} > c_{s_2}$

In this case, the call connection cost (c_e) is higher in one of the companies, but it has a lower cost of second after seconds full connection (c_s).

For the first t_e seconds of the communication, the company that offers lower cost for call connection will be more economical. To study those communications that last over seconds, it is necessary to analyze the second branch of the cost function. The lineal function for each company differs in the value of origin ordinate and slope, so at some moment both functions will have the same cost c(*t*). From this moment on, the company which was more expensive will become more economical. To know this point,

we propose:

$$c_{e_1} + c_{s_1}(t_i - t_e) = c_{e_2} + c_{s_2}(t_i - t_e)$$

For $t_i = \frac{c_{e_2} - c_{e_1}}{c_{s_1} - c_{s_2}} + t_e$. the cost of the communication for both companies is the same. Thus , $t_i > t_e$, since if the quotient sign $\frac{c_{e_2} - c_{e_1}}{c_{s_1} - c_{s_2}}$. For $c_{e1} > c_{e2}$ and $c_{s1} > c_{s2}$ or for $c_{e1} > c_{e2}$ and $c_{s1} > c_{s2}$, is always positive.

For those calls that last more than t_i, the most economical company will be the one that has the highest call connection cost.

Case 3: If $c_{e_1} > c_{e_2}$ and $c_{s_1} = c_{s_2}$ or if $c_{e_1} < c_{e_2}$ and $c_{s_1} = c_{s_2}$

In this case, for communication until seconds the company that has lower call connection cost will be more economical. For communications with duration higher than seconds, we consider the lineal function of the cost function . This function between these two companies differ in origin ordinate but they have the same slope, so the most economical company will be the one that has the lowest call connection cost.

Case 4: If $c_{e_1} < c_{e_2}$ and $c_{s_1} < c_{s_2}$ or if $c_{e_1} > c_{e_2}$ and $c_{s_1} > c_{s_2}$

In this case, for communications that last are under t_e seconds, the company which has higher call connection cost (c_e) will be the one which has higher communication cost.

For communications that last over t_e seconds, the cost functions involved differ in origin ordinate and slope, so it is necessary to know if there exists a point where the communication cost is the same for both companies. To know this point, we set out:

$$c_{e_1} + c_{s_1}(t_i - t_e) = c_{e_2} + c_{s_2}(t_i - t_e)$$

To $t_i = \frac{c_{e_2} - c_{e_1}}{c_{s_1} - c_{s_2}} + t_e$ the communication cost for both companies is the same. Here, $t_i < t_e$, because if the quotient sign $\frac{c_{e_2} - c_{e_1}}{c_{s_1} - c_{s_2}}$ is analyzed to $c_{e1} > c_{e2}$ and $c_{s1} > c_{s2}$ or to $c_{e1} > c_{e2}$ and $c_{s1} > c_{s2}$ is always negative. The two functions intersect in a point where $t_i < t_e$. This is impossible for the valuation type proposed. Therefore, after t_e seconds of call connection establishment, the communication will be more economical for that company which has lower seconds cost.

From this analysis it is worth asking:" How long t_e will this comparison go on?" It is necessary to consider the available credit to full consumption since some companies modify the call connection and second cost as more credit is required. This is the case of Company 1 plan. Thus, the comparison will be possible until the lower value between $\left\lfloor \frac{c_{r_1} - c_{e_1}}{c_{s_1}} + t_e \right\rfloor$ and $\left\lfloor \frac{c_{r_2} - c_2}{c_{s_2}} + t_e \right\rfloor$.

Another typical situation that can emerge of the above study is:

Situation 10

What is the cost of a second?

To answer this question, it is necessary to analyze the information given by the cell company instead of an immediate answer from the fee tables.

The average cost of t seconds call can be represented thus:

$$c_m(t) = \begin{cases} \frac{c_e}{t} & si\ 1 \leq t \leq t_e \\ \frac{c_e + c_s(t - t_e)}{t} & si\ \ t_e + 1 \leq t \leq \left\lfloor \frac{c_r - c_e}{c_s} + t_e \right\rfloor \end{cases}$$

$$Dom(f) = \left\{ t \in \mathbb{N} / 1 \leq t \leq \left\lfloor \frac{c_r - c_e}{c_s} + t_e \right\rfloor \right\}$$

The expression c_m (t) corresponds to a function defined by two branches. Each branch corresponds to rational functions.

The Study of situation 10 leads to the analysis of rational function. In particular, the study of what value the function tends to as time goes by. This implies the analysis functional asymptote and the deepening in the study of the relation between call duration and c_m (t). For this reason, the following situation is studied.

Situation 11

How do call duration and c_m (t) relate?

The average cost c_m (t) decreases for any time t of call duration because the call connection cost (c_e) and the second cost (c_s) are positive and $c_e > c_s$ t_e. This means that the longer the call, the lower the average cost.

For calls that last between 1 and t_e seconds, the function $c_m(t) = \frac{c_e}{t}$ is considered. Using the first derivate it is possible to verify that this function decreases as t increases. If t increased indefinitely, the average cost would be closer to zero.

However, this is limited for the companies study case because t E N so $1 \leq t \leq t_e$. On the other hand, the function has a vertical asymptote and this is verified resourcing to the study of function limit when tends to zero.

Using the same techniques as in the previous case, for calls longer than t_e the function $c_m(t) = \frac{c_e + c_s(t - t_e)}{t}$ has a horizontal aymptote y = c_s (This can be demonstrated by studying the function limit when $t \to \infty$)) This means that for communications longer than t_e seconds the price per second is close to (c_s).

The study does not stop here. The companies offer ways of payment such as prepaid or monthly fee. Likewise, same companies offer additional cost to fee depending on the call distance.

This study focuses on analyzing the call connection cost, however we should come back to the initial situation. It is necessary to suppose situations such as: If a person did not have cell company, which one should he choose? If a person had the cell company, should he opt for a better company or the plan within the same company?

As synthesis of the study I propose the following situation:

Situation 12

Cell company user is dissatisfied with his hired plan. Write a report that allows the user to take a decision bearing in mind his possible needs.

Conclusion

In this work, the essential characteristics of a didactic device are described. This didactic device allows to make gesture typical of the research pedagogy. The initial situation addresses a current situation that is of social interest. The study allows to re-find the usefulness of the different mathematical praxeologies and link them to the study of a real situation, giving answer to questions such as: How to use functions to real problems? How to choose the most economical fee to talk by cell phone? How to compare cell company fees? The study also allows to understand the ways cell companies charge and the fact that we can always talk t_e seconds because the companies charge this minimum fee although the communication is shorter.

In particular, the study of the initial situation leads to rediscover the functionality of mathematical praxeologies as: descriptive statistic, constant function, lineal function, rational function, function defined by branch, roof function, floor function, estimation by round and truncation, lineal equations and inequations, system of lineal equations, continuous and discrete of function, limit and continuity of function. The development of this didactic device in current conditions at high school in Argentine requires to go across praxeologies studied in different years and to introduce praxeologies that are absent in the curricular design.

In this work, the study of algebraic techniques was deepened. However, graphics and arithmetic techniques can also be applied. The graphic technique allows not only to contrast information obtained by arithmetic or algebraic techniques, but also to obtain information to start its interpretation that will be contrasted with other techniques after wards.

In relation with the gestures of the research pedagogy that may operate in the situation study, the following ones are the most relevant. The management of the didactic device can generate a rich set of questions, which is a fundamental gesture of research

pedagogy: The study and research dialectic. In this study, the formulation of questions has vital importance.

The media and middle dialectic is another gesture that could come into play with the proposal. The formulation of question by the community and the information provided by several media (mathematics book, web page of the companies, internet, etc.) will contribute to the middle and provide an answer to the initial situation.

In this study the dialectic of getting in-and-out of topic would also be present. So, giving answer to some questions requires to deepen several mathematical notions. Linked to this dialectic, it is necessary to analyze what knowledge is appropriate to solve the questions and how it is deepened. The grey level must be regulated because some praxeologies are studied without the depth expected by the scholar culture.

On the other hand, the teacher´s role is fundamental to progress and hold the problematization of the initial situation. The teacher must become omniscient so that the student can assume the responsibility of making questions and searching for the answers, proposing and studying concrete situations. The answers must be validated by the experience and knowledge of the students and the check between the work groups and the teacher. This addresses the individual and group dialectic.

Some experimentations of the didactic device with mathematics teacher training (Corica and Otero, 2016) give evidence about the conditions that affect the economy and ecology at current high school in Argentine.

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